

In the name of God



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***SECURITIZATION OF MORTALITY RISKS IN
LIFE ANNUITIES***

**A Thesis Submitted in Partial Fulfillment of the
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To my parents and kindly family

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Table of Contents

Abstract	7
Chapter 1: Securitization: Introduction and Theories.....	10
1.1 <i>Introduction</i>	10
1.2 <i>Relevance and importance of the thesis subject.....</i>	10
1.3 <i>Theoretical foundation(s) of the thesis.....</i>	11
1.4 <i>Main hypothesis of the thesis.....</i>	12
1.5 <i>Methodology.....</i>	12
1.6 <i>Data requirements.....</i>	12
Chapter 2: Literature Review.....	14
2.1 <i>The Importance of Studying Mortality Changes on Life Insurance</i>	14
2.2 <i>Mortality Risk Modeling.....</i>	15
2.3 <i>People and Future Life Times.....</i>	19
2.3.1 <i>Longevity Risk.....</i>	19
2.3.2 <i>Life Annuity.....</i>	19
2.4 <i>Securitization Era.....</i>	20

2.4.1	<i>Definition of Securitization</i>	21
2.4.2	<i>Securitization of Catastrophe Mortality Risks</i>	22
2.5	<i>Using mortality bonds to hedge aggregate mortality risk</i>	24
Chapter 3:	<i>Securitization of Mortality Risk</i>	29
3.1	<i>Introduction</i>	29
3.2	<i>overview of securitization</i>	31
3.2.1	<i>The Structure of Asset-Backed Securities</i>	32
3.2.2	<i>Drivers of Demand for Securitization</i>	34
3.2.3	<i>Special Purpose companies</i>	35
3.3	<i>Securitization in the Life Insurance Industry: General Considerations</i>	37
3.3.1	<i>Longevity risk</i>	37
3.3.2	<i>Stakeholders in Markets for Mortality-Linked Securities</i>	40
3.3.3	<i>Mortality – Linked Securities</i>	42
3.3.3.1	<i>Longevity Bond</i>	42
3.3.4	<i>Candidates for Securitization</i>	45
3.4	<i>Existing Mortality – Linked Securities</i>	49
3.4.1	<i>Design of the Swiss Re bond</i>	49
3.4.2	<i>Design of the EIB longevity bond</i>	51
3.5	<i>Example</i>	52
3.6	<i>Wang Transform</i>	55
3.6.1	<i>The Normal Distribution</i>	55

3.6.2 Standard Normal Distribution.....	57
3.6.3 A Universal Pricing Method.....	57
3.6.4 Unified Treatment of Assets & Losses.....	59
3.6.5 Distortion Operators.....	59
3.6.6 A New Distortion Operator.....	63
Chapter 4: Measuring Longevity Risk	65
4.1 Introduction	65
4.2 Theoretical Considerations.....	65
4.3 Why transform?.....	68
4.4 Equivalent Transformation Forms.....	70
4.5 Preserving Normal and Lognormal Distribution under Wang Transformation.....	70
4.6 Market price of risk.....	73
4.7 Data Consideration.....	77
Chapter 5: Conclusions and Suggestions.....	78
Appendix A. Tables.....	81
References.....	84

Abstract

Insurers have in the past few decades faced longevity risks - the risk that annuitants survive more than expected - and therefore need a new approach to manage this new risk. In this dissertation we survey methods that hedge longevity risks. These methods use securitization to manage risk, so using modern financial and insurance pricing models, especially Wang transform and actuarial concepts, we will show how bond contracts, provide hedges that insurers need.

Securitization is one of the most important innovations of modern finance. Securitization, the trading of cash flow streams, enables the parties to the contract to manage and diversify risk, to take advantage of arbitrage opportunities, or to invest in new classes of risk that enhance market efficiency. The cash flow streams to be traded often involve contingent payments as well as more predictable components which may be subject to credit and other types of counterparty risk.

What we intended to do was assessment of the possibility of designing longevity risk bond to cover this risk for annuity writers in Iran. After applying Wang transform to existing life table data, we got a risk parameter that cause the transformed survivor curve lies below the given table. This means that there is no longevity risk for annuity writers in Iran, and hence there are no significant reasons to design and publish longevity bond. Of course this result does not necessarily mean that insurance companies encounter other side of mortality risk, i.e. the insureds die sooner than expected, which encourage them to design and sell mortality bonds.

So comparing our findings and other studies, the suggestion of this thesis is making a real life table. This help insurers and annuity writers to assess their longevity risk precisely, and manage the hedge they will need.

Chapter 1

Securitization: Introduction and Theories

1.1 Introduction

Our objective in this chapter is to provide the importance and relevance of the thesis subject along with theoretical foundation and thesis hypothesis, all to respond to the question that whether the longevity risk, the survival probabilities of annuitants are higher than expected, exist or not.

1.2 Relevance and importance of the thesis subject

As life expectancy in the world in recent years has improved, and due to uncertainty in mortality forecasts, insurers have found out that management of longevity risk - the risk that annuitants survive more than expected - is more important than ever. According to Mitchell, Poterba, Warshawsky, and Brown (2001), an individual annuity contract is a more attractive product now than ten years ago, and Social Security reform should increase demand for individual annuity product in the future. As demand for individual annuities increases, insurer's need for risk management of potential mortality improvements increases as they write new individual annuity business.

1.3 Theoretical foundation(s) of the thesis

Longevity bonds are securities that are published in capital market, to cover insurance annuity writer's and pension fund's longevity risk. As we represent later in this chapter, we are going to find out if this risk exist in Iran or not. We are going to apply Wang (1996, 2000, 2001)'s method to pricing longevity bond risks, which unifies financial and insurance pricing theories. If $\Phi(x)$ is the standard normal cumulative distribution function with a probability density function

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \quad (1.1)$$

For all x , then Wang distortion operator is given as

$$g_\lambda(u) = \Phi[\Phi^{-1}(u) - \lambda] \quad (1.2).$$

Where λ is the market price of risk. Considering insurer's liability X over a time interval $[0, T]$, the value or fair price of the liability is the discounted expected value under the distribution obtained from distortion operator. The formula for price is as follow

$$H(X, \lambda) = E^*(X) = \int x dF^*(x) \quad (1.3)$$

Where $F^*(x) = g_\lambda(F)(x) = \Phi[\Phi^{-1}(F(x)) - \lambda]$. The parameter λ is denoted as the market price of risk, reflecting the level of systematic risk. So, if cumulative density function for an insurer's liability X , is F , the Wang transform will produce a "risk-adjusted" density function F^* . $E^*[X]$ is the mean value under this new function, which will define a risk-adjusted "fair value" of X at time T , which can be further discounted to time zero, using the risk free rate. Finally what we want to do, is to use

pseudo-observed annuity prices to estimate the market price of mortality risk, then use the same distribution to price mortality bonds.

1.4 Main hypothesis of the thesis

Iranian insurance companies faced longevity risk.

1.5 Methodology

At first we start with an example and use actuarial and derivatives concepts to describe cash flows between special purpose contracts SPC, investors and insurers. Then we introduce and explain bond contracts as hedge tools. Next step is introducing Wang transform for pricing longevity risk. In this case we use standard normal distribution in defining the Wang distortion operator and then estimate parameter λ as the market price of risk, which is used in Wang transform method, using TD 1988 life table and risk free rate of interest, 15%. We use binomial distribution for ℓ_{x+t} , the number of survivors from ℓ_x who survives to age $x+t$. because we have ℓ_x values, so ℓ_{x+t} has approximately normal distribution.

1.6 Data requirements

In the absence of marketed annuity products in Iran, we use DT 1988 life table which is being used by Iran Central Insurance and most of insurance companies in Iran. Interest rate we use is risk free interest rate 15% which Iranian insurance companies

use in life insurance and other securities are designed according to this risk free interest rate.

Chapter 2

Literature Review

2.1 The Importance of Studying Mortality Changes on Life Insurance

There are few researchers who investigate and scrutinize the issue of natural hedging. The majority of the previous and earlier studies explores the impact of mortality changes on life insurance and annuities separately, or looks into a simple composition of life and pure endowment life contracts (Frees et al., 1996; Marceau and Gaillardetz, 1999; Milevsky and Promislow, 2001; Cairns et al., 2004). The focal point of researches on the impact of mortality changes on life insurance are concentrated on “bad” shocks, while those on annuities are concentrated on “good” shocks.

However, life insurance and annuity mortality experience can be very various and distinct, thus there is “basis risk” included in utilizing annuities to hedge life insurance mortality risk. Consequently, the immunization model which was proposed by Wang, Yang, and Pan cannot pick up this factor. The proposed model hedge risks based on the mortality experience which was derived from the researcher’s analysis on the impact of the changes of mortality factors in Taiwan (Wang et al, 2003).

In addition, Marceau and Gaillardetz “examine the calculation of the reserves in a stochastic mortality and interest rates environment for a general portfolio of life insurance policies”. According to their numerical examples, they use portfolios of term life insurance contracts and pure endowment polices and mainly focuses on

convergence of simulation results. It is noticed that there is a hedging effect in their obtained results, but they do not follow up and continue the issue (Marceau and Gaillardetz, 1999). Also, Cox and Lin show that “natural hedging utilizes the interaction of life insurance and annuities to a change in mortality to stabilize aggregate cash outflows”. In this regard, their empirical evidences propose natural hedging as the leading and prominent factor contributing to annuity price differences after they control for other variables. These differences become more considerable and momentous for those insurers selling relatively more annuity business. However, they anticipate that life insurers may achieve or accomplish the same consequence (H. Cox and Lin, 2005).

2.2 Mortality Risk Modeling

“Over the past half century, and especially in the most recent decades, remarkable mortality improvements have led to the growth of the population of older people” (Bourguignon and Morrisson, 2002; Lakdawalla and Philipson, 2002; Vaupel, 1998). Since this progression is unanticipated, thus, it has a negative and dissenting impact on pension plans and annuity providers simultaneously. “In the US, private defined benefit pension plans currently have close to \$6 trillion in liabilities for future benefits. In addition, US life insurers hold approximately \$2 trillion in annuity reserves” (Salou and Hu, 2006; ACLI, 2006). Whereas annuity benefits may require to be paid longer than anticipated, hence, uncertainty/hesitancy of longevity progressions augments risk for two categories which are included pension funds and annuity insurers.

“In a recent study of pension liabilities of the companies in the UK's FTSE100 index, found overly optimistic longevity assumptions for pension valuations reported at the end of 2007” (Cowling and Dales, 2008). In comparison, in modeling life insurance, one naturally looks at scenarios which are pessimistic around mortality progression and mostly involve threats such like epidemics. Many several latest articles concentrate on such mortality risks management. “Some public health experts think that a pandemic is overdue and another will inevitably occur” (Dowdle, 2006). “Should a pandemic occur, a life insurer will suffer financially since it will pay more death benefits than expected when the policies were issued”. Toole had conducted a research around this issue more precisely and concluded that “the industry as a whole can withstand a severe pandemic, as severe as the 1918 pandemic, with a loss of about \$64 billion relative to aggregate risk-based capital (RBC) of about \$256 billion in 2005” (Toole, 2007). As he notes, if a severe pandemic were to take place when the financial markets are not strong, then the following factors can be occurred, they are as follows:

1. Financial impact could be much worse.
2. The number of companies close to insolvency could be very vast.

Therefore, it seems obvious that involving pandemic impacts is a considerable and significant issue in modeling mortality for life insurance liabilities.

In light of the above discussion, the terms mortality risk and longevity risk are opposites according to Toole. He believes that mortality risk is “the risk of more deaths than expected or the risk that observed death probabilities are higher than expected”. On the other hand, “more people may survive than expected or observed death rates may be lower than expected”. A surge in mortality progression in a short period is

driven from unexpected improvements in medicine and health technology which may give rise to longevity risk (e.g. one or two years) or extra progression over a long period (e.g. more than ten years). “Actual mortality has been improving, so parametric models estimated with actual data will reflect improving mortality. Models that have a random deviation from the expected mortality may reflect some longevity risk, but this is not a reflection of a fundamental change in the trend (as we have in mind)”.

Many researchers concentrate on mortality risk as deviations from a trend. In those papers, the base trend may represent mortality progressions, but longevity risk is not modeled obviously. On the other hand, pension and annuity study has more concentrated on longevity risk. Loise and Serant model longevity and mortality risk utilize a stationary Gaussian process (Loise and Serant, 2007). However, this model may not be proper for modeling longevity (and mortality) shocks which may not pursue a stationary Gaussian process. To illustrate the stochastic longevity trend, Hári and his colleagues extend “the Lee_Carter model with a time-varying, stochastic drift” (Hári., 2008). Biffis (2005) “captures mortality random departures around a time-varying target with a longevity compound Poisson process, but that model cannot guarantee a nonnegative force of mortality”. Specifically, we survey a new approach by presenting a trend reduction jump component to illustrate longevity risk. Unexpected longevity improvement, in general, may be less dramatic than that of a mortality death shock, but in the long run longevity risk may be just as important. Most longevity risk events in the past seem to have a pattern; unexpected survival gains often extended over a long period of time, leading to a steeper downward sloping force of mortality curve. “The traditional one-time jump models such as the model that

combines a geometric Brownian process and a compound Poisson jump process cannot provide this kind of longevity risk". Furthermore, to gain relevant consequences, a stochastic mortality model must represent three leading and essential factors, they are as follows:

1. both mortality improvement and deterioration jump factors;
2. correlation among different ages and over time;
3. and uneven effects of a mortality jumps across different ages.

To sum up, the model attempts to address the above-mentioned issues. In particular, "we are explicitly combining both mortality and longevity risk in a single, comprehensive model in order to make a more realistic assessment of future survivor dependent cash flows". Finally, we indicate how to specify a parsimonious version with historical data. "Derivatives can be written on indices based on publicly available data like the Life Metrics index (Coughlan et al., 2007) offered by Goldman_Sachs and the Credit Suisse index".

Friedberg and Webb at 2005, used the Lee_Carter model to demonstrate that longevity risk is necessarily uncorrelated with the return on the "market portfolio" as measured by the S&P500. Applying the capital asset pricing model, these Authors argue that "it should be possible, at least in theory, to transfer the longevity risk to the financial market at very low cost and it seems indeed that several tentative to perform such a transfer have already been observed on markets". The European Investment Bank called Banque Nationale de Paris issued the first longevity bond in November 2004 in France. Since then, financial institutions such as Swiss Re. has also started to trade

mortality derivatives over-the-counter, so that a wholesale market in aggregate mortality risk could emerge in the future.

2.3 People and Future Life Times

Creighton and his colleagues have provided data which illustrate that people are living longer and more are retiring at younger ages (Creighton et al., 2005). In this respect, some other researchers believed that individual future lifetimes are also becoming more variable and changeable (Booth et al., 2002; Morgan, 2007). To sum up, the future life time will result in “an increased reliance on income sources including life annuities and lifetime income guarantee products to fund longer retirement time periods” (Creighton et al., 2005; Lin and Cox, 2005).

2.3.1 Longevity Risk

Longevity risk is the other side of mortality risk. Although mortality improves over time, future rates of improvement are uncertain (Lin and Cox, 2007).

If future mortality improves relative to current expectations, life insurer liabilities decrease because death benefit payments will be later than expected. However, annuity writers have a loss relative to current expectations because they have to pay annuity benefits longer than expected (Lin and Cox, 2005). This is what is named longevity risk, the risk that individuals live more than expected.

2.3.2 Life Annuity

One contingency facing individuals is how long they will live and whether their finite retirement funds will be sufficient to finance consumption needs over a possibly long remaining lifetime. The insurance product that has arisen in response

to this contingency is the *life annuity*. Like the progression of health insurance coverage, insurance companies originally issued fixed life annuities to individual.

In reality, neither individuals nor insurance companies know exactly how long an individual annuitant will live. In the case of an annuity bond that continues to pay out coupons for as long as the individual is alive, its price depends on the whole probability distribution of death rates for this individual, in other words, T , individual lifetime, is a random variable and not a fixed parameter. (Blake and Burrows, 2001)

2.4 Securitization Era

“The securitization era began in the 1970s with the securitization of mortgage loans by the government sponsored enterprises (GSEs) Fannie Mae, Ginnie Mae, and Freddie Mac, which were created by the Federal government with the objective of facilitating home ownership by providing a reliable supply of home mortgage financing”. The second main and major improvement in this field was the presentation of asset-backed securities (ABS) based on other types of assets. It is worthy of attention that the first U.S insurance securitizations occurred in 1988 and included sales of rights for emerging profits from blocks of life insurance policies and annuities (Millette et al., 2002). Furthermore, insurance linked securitizations speed up during the 1990s with the development of catastrophic risk (CAT) bonds and options and an increasing volume of life insurance and annuity securitizations. Cowley, Brothers, and Cummins conclude and state that securitization has the possibility as well as the potential to enhance the efficiency and profitableness of insurance and financial