INSURANCE PRICING
BASED ON OPTION PRICING METHODS

By

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Abstract

We have applied an insurance pricing methodology which solves the problem of previous insurance pricing methods that do not consider the effect of firm's insolvency risk. This model has enabled insurance firms to price insurance by line in a property-liability insurance companies with multiple line of business considering the insolvency risk. In this framework, the fair insurance price is determined by the liability payoff distribution, the arbitrage-free valuation function, and the contract's safety level which is measured by the value of the insolvency put option. Using option pricing methods, we derive a closed form solution for insolvency put option and further, predict that the price of insurance is inversely related to the overall insolvency risk of insurance firms. Finally, we have illustrated a numerical example in the single period discrete state to show how to allocate the insolvency option to lines of business determined by the outstanding liability and show that this allocation depends on the insure total capital. Also we have written a file in the *Mathematica Programming Language* for the practical purpose in the continuous state model.

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Introduction

In standard capital budgeting each project is determined by its cash flows and discount rate which dose not depend upon the characteristic of the investors. Regulators view capital from the prospective of solvency and rating agencies look at the balance more from a debt holder's standpoint. In contrast, insurance companies manage their capital to optimize their reinsurance decision where reinsurance is an alternative source of protection for insurance companies. In common, capital allocation used as a tool by insurers to manage their underwriting risk. They view capital in terms of running their business effectively as viewed from the shareholder's perspective. Financial institutions such as banks and insurance industries face frictional costs of holding capital. Because holding capital is costly in order of factors such as corporate taxation, regulatory costs and agency costs, financial institutions optimally do not hold sufficient capital to completely protect their operation from unexpected events that deplete capital. Banking and insurance relationships often involve risk transfer and risk management and investors of this institutions are more concerned about insolvency risk than investors of non-financial firms. Also capital allocation theories recognize that risky businesses contribute more to insolvency risk than lower risk activities. Capital allocation is useful in achieving the goals of competitive pricing of

insurance contracts and making optimal capital budgeting decision.

The interconnection of financial and insurance markets has become more coherent during the past two decades. The convergence of insurance and finance has considered in its most advanced level in ***Alternative Risk Transfer Methodology*** (**ART**), where insurance and financial risks are covered jointly. In this case, **insurance pricing based on option pricing** techniques represents a significant breakthrough over **traditional actuarial pricing** models because it recognizes that insurance pricing should be consistent with the wealth pricing model and avoid arbitrage opportunities. Traditional actuarial pricing is based on real probabilities P, which reflects the actual likelihood of loss event as follow:

$$Premium = \frac{1}{1+r_L}\ E^P[L] + \theta[L]$$

where $E^P[L]$ is the expectation of the loss event under probability measure P, r_L is the growth rate of loss and $\theta[L]$ is the safely loading or risk premium that reflects the risk related to the each contract and expenses. In finance, the non-arbitrage pricing principal applies equivalent martingale measure Q where the price of an uncertain cash flow is determined by

$$price = \frac{1}{1+r}\ E^Q[L]$$

where $E^Q[L]$ is the expectation of an uncertain cash flow under measure Q and r is the free-risk interest. Consequently non-arbitrage pricing is not modification due to a safely loading. Actuarial pricing approach argues that price of insurance is not based on an assumption of hedging and so, there is not any liquid secondary market for insurance and reinsurance contracts. The relevance of option pricing relies on liquidity and efficiency of the insurance market. Liquidity and efficiency are not features of the insurance market in general and hence the non-arbitrage approach to pricing of

insurance may be inadequate . ***Mildenhal*** [16] and ***Wange*** [29] argue that there are subtle differences between option pricing and actuarial pricing. In 2002, *Wange* introduced a new transform and correlation measure that extends ***Capital Asset Pricing Model*** **(CAPM)** to pricing all kinds of assets and liabilities, whether is traded or is underwritten in finance or insurance. He illustrates the application of the new framework to insurance company capital allocations, and to the determination of fair values of insurance liabilities.

As it was mentioned, generally option pricing approach to pricing of insurance may be inadequate until liquidity and efficiency are not features of the insurance market. However, this situation is different when insurance company applies ART techniques to optimize their reinsurance decision. Indeed, where insurance risk is traded or transfered to the financial markets, the price of insurance should reflect the ***insolvency risk***. Consequently,

> **The option pricing approach may be limited to traditional insurance structures with only limited exposure to financial risk, but the market price is relevant to insurance structures whose performance heavily depend on the performance of financial market.**

In this thesis we consider insurance pricing with respect to each line and the entire amount of insolvency risk in the multiple line property -liability insurance company that its performance significantly depends on the performance of financial market and hence,

> **option pricing model is preferred to the traditional actuarial pricing approach because traditional actuarial pricing does not**

recognize the free-arbitrage assumption and insolvency risk which is involved with insurers who enter financial markets.

Since allocation of capital including an insolvency risk is one of the important issue in the insurance companies, various methodologies have been demonstrated to introduce a system of capital allocation such as ***Merton and Perold*** [14], ***Panjer*** [20], ***Michale Sherris and John van der Hoek*** [24], and ***Cummins*** [2]. An important early paper on capital allocation applying insolvency risk is ***Merton and Perold*** [14]. They consider a firm with N lines of business and calculate the market value of insolvency put value. The principal problem with their methodology is that their model dose not allocate the entire firm's capital. Indeed, their model is appropriate when considering merges of entire divisions or lines of business and is less efficient when considering the pricing of individual products such a bank loans or insurance policies which represent marginal changes in the composition of the firm. ***Stewart C.Meyrs and James A.Read,Jr.***[18], apply a unique capital allocation method for pricing insurance contracts which requires allocation of capital to lines of business. They suggest that competitive premiums depend on total capital requirements and on their allocation to lines of insurance and consequently, if capital allocation is wrong, cost allocation will be wrong too. They introduce a marginal capital allocation model that allocates the entire firm's capital. They consider a firm with N lines of business and take derivative of the firm's overall insolvency put value with respect to loss liabilities of each of the N lines. Their approach is not dependent on any set of distributional assumptions with respect to the firm's asset or liability. However, they derive the model under the implicit assumptions that assets and liabilities are jointly lognormal. This assumption involves modeling the firm as a

Black-Scholes option, where total assets and liabilities are distributed as lognormal distribution. ***Kneuer***[8], ***Ruhm and Mango*** [22], ***Vrieze***, and ***Brehm*** [28], and ***Mildenhall*** [17] analyze the practical limitations and the distributional hypothesis under Myers and Read approach. ***Meyers***[15], argues that if no tax or other costs of holding capital are into account, capital allocation is not necessary for the case of expanding or contracting lines of business which was argued by ***Phillips, Cummins and Allen*** [21], too. They argue that it is not appropriate to allocate capital by line where the price of insurance by line is determined by the overall risk of the firm and the line specific liability growth rates. ***Venter***[26]-[27], claims that Myers and Read approach will not give perspicuous guidance about the profitability of different lines of business and making capital budgeting decisions, but believe the method is appropriate for the purpose of pricing insurance contracts. ***Helmut Gründl*** and ***Hato Schmeiser*** [5], show that capital allocation to lines of business based on the Myers and Read approach, in the case of no frictional cost, is not necessary for insurance rate making and may lead to incorrect loading. ***Michael Sherris*** [23] shows that allocation of capital to lines of business requires an allocation of assets by lines and there is no unique or optimal way to do this in a complete market's model with no friction cost. He also shows that allocation of insolvency put option depends only on the insurer total capital and not the allocation of capital to line of business.

To measure the price of insurance, we have applied the ***Economic Premium Ratio (EPR)*** which is the standard price measure in insurance and is suggested by ***Winter*** [30]. *EPR* is defined as the ratio of the premium revenues net of expenses and policyholders dividends for a given insurer and line of insurance to the estimated present value of the losses for the line. Hence, *EPR* provides a measure of the insure's

return for underwriting a line of business. In this framework, we have determined the fair insurance price by the liability payoff distribution, the arbitrage-free valuation function and the contract's safely level which is measured by the value of the insolvency put option. We have considered a multi-line insurer and no allocation of equity capital to lines of business which is consistent with *Phillips, Cummins and Allen*(1998). In general, this thesis has theoretically proved that price of insurance depend upon firm's capital structure . All the theoretical proofs under this work has practically proved by ***Cummins, Lin, and Phillips*** [3]. They practically test our theoretical prediction by analyzing the prices of insurance risks for **U.S. property-liability insurers** over the period **1997-2004**. To test the theoretical hypothesis, they estimate the price of insurance by line, the variance and covariance of insurer asset and liability portfolios, the firm's overall insolvency risk and the marginal contributions of lines of business to insolvency risk as an input variables. For estimating the price of insurance they use *EPR* that is consistent whit the price of insurance we have considered in this work. For estimating the growth rate of liabilities in each line r_{L_i}, they follow *Phillips, Cummins and Allen*(1998) and consider the average five year growth rate of total industry accident year losses and loss adjustment expenses incured for each line of business. Finally, they practically test our proved theoretical propositions by considering regression analysis. Their consequences strongly support our theoretical proofs. Another empirical work in this area has done by *Phillips, Cummins* and *Allen* [21]. Their sample includes ninety publicly traded insurers over 1988-1992 that obtained from the NYSE/AMEX and NASDAQ CRSP taps. Their statistical sample consists of the property-liability insurance companies or multi-line

insurers with at least 25 percent of their premium revenues in property-liability activity. The consequences they obtain is consistent with our theoretical predictions.

Restrictions involved for the empirical test on the Iranian Insurance Market:

Central Insurance of Iran (Bimeh Markazi Iran) was established in 1971 by the Act of Parliament for the purpose of regulating, expanding and guiding Insurance Industry in Iran. The insurance service in Iran is also presented by insurance agents and brokers both of which are authorized to act in the market after passing the relevant tests and receiving the license form Bimeh Markazi Iran. Under the Iranian compulsory reinsurance regulations, article 71, all insurance companies which operate in Iran must insure with the Central Insurance of Iran 50 percent of their direct insurance business in the field of life insurance, and 25 percent of their direct insurance in other fields. **Note:** what insurance companies accept under the heading of reinsurance is not subject to the provisions of this Article.

Also according to article 73 of Bimeh Markazi Iran's regulations, insurance companies operating in Iran shall be obliged to reinsure with Central Insurance of Iran under the same terms and conditions as they effect reinsurance abroad, the equivalent of 30 percent of the balance of the insurance effected directly by them in excess of the compulsory reinsurance. The insurance companies in Iran are active in life and non-life fields **according to the tariffs** which are approved and ratified by High Council of Insurance and unfortunately, they are not based on accurate actuarial calculation, loss distribution or real probability. According to the Iranian Insurance Act, the retention of insurer in the case of direct risk can not exceed 20 percent of the paid-up

capital plus statutory and technical reserves minus unpaid losses, and it reduces to 10 percent in the case of indirect risk. Therefore, under this law, **Iranian insurance companies do not have any choice for optimize their reinsurance strategy by (ART)**. Under this situation, the remaining amount of premium that is collected by property-liability line is not remarkable to attract insurance company to invest in the stock market. Another restriction is about investment strategy. According to the article 42, investment regulation, the insurance company that is active in the non-life fields can invest regulated amount in the **governmental guarantied bonds and safety deposits** and investment in the stock market has some restrictions which all the above activities are supervised by Bimeh Markazi Iran. For example Saman Insurance Company and Parsian Insurance Company, the Iranian privet insurance companies, invest retention of their liability-property premiums in safely deposit and just a little amount of their paid up capital is invested in the Iranian Stock Exchange Market. **So this model cannot be used in the Iranian insurance market where our model is relevant for insurance companies that have highly exposer to financial risk**.

In this thesis we make the following assumptions:

- Financial loss events are risky, so insurers face significant insolvency risk and the risk of insolvency matters to consumers.
- There is not any frictional costs such as corporation taxation and agency costs.
- Insurance companies do not have any reinsurance strategy.
- **Equal Priority rule** is considered in the case of bankruptcy.
- Perfect market conditions is held under mathematical operations.

- Premiums and liabilities accounts follow joint lognormal distributions.

Applying option pricing techniques and considering the set of mentioned assumptions:

1. We will show that insolvency put option is consistent with the standard Black-Scholes formula where underlying asset is the asset-to-liability ratio and it depends on the

    - firm's total assets and liabilities,
    - the risk-free interest rate adjusted of the growth rates of the insurer liabilities,
    - and, time untile payment of loss liabilities.

2. We have mathematically proved that in a competitive and efficient insurance market, the price of insurance will be inversely related to the firm's insolvency risk.

3. We prove that price of insurance depends on the total firm's capital and it is not appropriate to allocate capital by line.

4. We show that the expected sign of growth rate in liabilities in a given line of business is ambiguous.

This work has been organized in the following manner: In the first chapter, we have focused on a list of mathematical definitions to describe the financial model. In the second chapter, we have dealt with the construction of **Itô Integral** and have introduced Itô calculus. The third chpter of this project has provided a self-financing

portfolio and derived a financial model for the price and wealth process. Then, it has been followed by representing an important financial concept named completeness of the market and its property. In the forth chapter, we have applied theory of martingales and stochastic partial differential equation to show the mathematical framework of analysis of option contracts. Then, we have derived the **Black-Scholes equation** and proved **Girsanove's Theorem** which gives an alternative interpretation of the Black-Scholes formula as an expectation under the changed distribution. Chapter five has introduced a link between insurance pricing and financial calculations. We have adopted the option pricing approach to model insurer's default risk. We have demonstrated a model based on joint lognormal distribution in the multi-line insurance companies. We then have formulated the possible situation of claims payments and derived a closed form solution of insolvency put option and further written a *Mathematica Package* for practical use.