IN THE NAME OF GOD

SOLUTION OF TRANSONIC & SUPERSONIC INVISCID FLOW EQUATIONS IN 3-D ECCENTRIC NOZZLES

BY

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To: My Mother and Father

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ABSTRACT

"SOLUTION OF TRANSONIC & SUPERSONIC INVISCID FLOW EQUATIONS IN 3-D ECCENTRIC NOZZLES"

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A computer program for solving the inviscid flow equations in three-dimensional eccentric nozzles as well as concentric nozzles is developed. The program uses the cell-centered finite-volume method based on Roe's approximate Riemann solver scheme. To show the accuracy and capability of this code, the results of concentric circular nozzles are first compared with simple one-dimensional analytic solution, and then the results for steady and unsteady flow through eccentric and concentric convergent-divergent nozzles are presented. The results are given for various area and pressure ratios, and different values of the inlet Mach number.

TABLE OF CONTENTS

Content	Page
ABBREVIATIONS AND SYMBOLS	**vii**
LIST OF FIGURES	**ix**
CHAPTER I : Introduction	**1**
1-1 Artificial Viscosity	1
1-2 Upwind Schemes	3
1-3 Godunov-Types Methods	6
1-4 Roe's Approximate Riemann Solver	9
CHAPTER II: Theory	**19**
2-1 Conservation Form	21
2-2 Generalized Curvilinear Coordinate Transformations	22
2-3 Discretized the Euler Equations	28
2-3-1 Finite Volume Structure	29
2-3-2 Derivation of the Cell-Faces Normal	31
2-3-3 Flux Difference Splitting Algorithm	31
CHAPTER III: Numerical Method	**33**
3-1 Explicit Upwind Scheme based on Roe's Method	33
3-2 Grid Generation	35
3-3 Boundary Conditions	36
3-3-1 Inlet and Outlet Boundary Conditions	38
3-3-1-1 Subsonic/Supersonic Inflow	38
3-3-1-2 Subsonic/Supersonic Outflow	39

3-3-2 Solid Wall Boundary Condition 41

CHAPTER IV: Results **43**

4-1 Concentric Nozzles 43

4-1-1 Axisymmetric Flow 44

4-1-2 Three-Dimensional Flow 47

4-2 Eccentric Nozzles 64

4-3 Conclusion and Recommendation 83

APPENDIX A **84**

APPENDIX B **85**

REFERENCES **87**

ABSTRACT AND TITLE PAGE IN PERSIAN

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ABBREVIATIONS AND SYMBOLS

A	wave speed
a,b,c,d	cell-face points
A,B,C	Jacobian of flux vector with respect to conservative variables E,F,G, respectively
c	speed of sound
e	internal energy per unit mass
E,F,G	column-vector of Cartesian flux functions
h	enthalpy per unit mass
j,k,I	computational coordinate indices
J	Jacobian of coordinate transformation
λ,r	left and right eigenvectors
L, L^{-1}	left and right eigenvector matrices
M	matrix of non-conservative variables
p	pressure
Q	column-vector of conservative variables
r, θ,x	circular cylindrical coordinates
R	local radius of nozzles
u	scalar dependent variable
u,v,w	velocity component in x,y,z directions
U,V,W	contravariant velocities
$V_{prim.}$	column-vector of primitive variables
W	column-vector of characteristic variables
x,y,z	Cartesian coordinates

Greek Symbols

α	pseudocharacteristic variables
δ	central difference operator: $\delta u_i = u_{i+1/2} - u_{i-1/2}$

δ^+	forward difference operator: $\delta^+ u_i = u_{i+1} - u_i$
δ^-	backward difference operator: $\delta^- u_i = u_i - u_{i-1}$
Δ	mesh size in the given directions
Δt	time step
γ	specific heat ratio
$\kappa_{x,y,z}$	x,y,z components of cell face normals
λ	eigenvalue
Λ	diagonal matrix whose diagonal elements are eigenvalues
θ	$(u^2 + v^2 + w^2)/2$
ρ	density
τ	transformed time coordinate
Ω	volume
$\partial\Omega$	curve surface
ξ, η, ζ	transformed coordinates

Superscripts

n	iteration level or time level
$(\bar{\ })$	fluxes in the transformed domain
$(\hat{\ })$	numerical flux of cell face

Subscripts

0,1,2	subscript 0 indicates a wall point and subscripts 1 and 2 indicate above the wall point
j,k,i	mesh points location
x,y,z	partial differentiation with respect to x,y,z
ξ, η, ζ	partial differentiation with respect to ξ, η, ζ

LIST OF FIGURES

Figure **Page**

Figure 1-1: Step One: Piecewise Constant Distribution at $t = n\Delta t$ 7

Figure 1-2: Step Two: Exact Resolution of Riemann Problem at Interfaces 8

Figure 1-3: Step Three: Godunov's Method for the Linear Wave Equation with a>0 9

Figure 1-4: Relations Between the Conservative Variables Q, the Primitive Variables $V_{prim.}$ and the Characteristic Variables W 11

Figure 1-5(a): Eccentric nozzle geometry in x-z plane 17

Figure 1-5(b): Eccentric nozzle geometry in y-z plane 17

Figure 1-5(c): Eccentric nozzle geometry in x-y plane 17

Figure 2-1: Region of Dependence and Zone of Influence of point p for a Hyperbolic Problem with Two Characteristics per Point 20

Figure 2-2: Characteristic Lines for a One Dimensional Flow 28

Figure 2-3: Computational Finite Volume Cell 29

Figure 3-1: Two Eccentric Circles 35

Figure 3-2: A Typical Sectional Grid 37

Figure 3-3: Characteristic at Subsonic Inflow and Outflow Boundaries 39

Figure 4-1: Mach contour for: Area Ratio=2.0,Pi/Pe=100. 49

Figure 4-2: Pressure contour for: Area Ratio=2.0, Pi/Pe=100. 49

Figure 4-3:Centerline Mach distribution for: Area Ratio=2.0, Pi/Pe=100 50

Figure 4-4: Mach contour for: Area Ratio=2.0, Pi/Pe=3.0 51

Figure 4-5: Pressure contour for: Area Ratio=2.0, Pi/Pe=3.0 51

Figure 4-6: Centerline Mach distribution for: Area Ratio=2.0, Pi/Pe=3. 52

Figure 4-7: Mach contour for: Area Ratio=2.0, Pi/Pe=10.0 53

Figure 4-8: Pressure contour for: Area Ratio=2.0, Pi/Pe=10.0 53

Figure 4-9: Centerline Mach distribution for Area Ratio=2.0, Pi/Pe=10. 54

Figure 4-10: Mach contour for: Area Ratio=1.2, Pi/Pe=20.0 55

Figure 4-11: Pressure contour for: Area Ratio=1.2, Pi/Pe=20.0 55

Figure 4-12: Centerline Mach distribution for Area Ratio=1.2,Pi/Pe=20. 56

Figure 4-13: Mach contour for: Area Ratio=1.2, Pi/Pe=40.0 57

Figure 4-14: Pressure contour for: Area Ratio=1.2, Pi/Pe=40.0 57

Figure 4-15: Centerline Mach distribution for Area Ratio=1.2,Pi/Pe=40. 58

Figure 4-16: Mach contours for: Area Ratio=1.56, Mi=1.25, Pe/Pi=2.0 59

Figure 4-17: Pressure contour for: Area Ratio=1.56, Mi=1.25, Pe/Pi=2.0 59

Figure 4-18: Centerline Mach distributions for 1-D and 3-D Flow: Area Ratio=1.56, Mi=1.25, Pe/Pi=2.0 60

Figure 4-19(a): Mach contours for: Area Ratio=2.0, Pi/Pe=2.8 61

Figure 4-19(b): Mach contours for: Area Ratio=2.0, Pi/Pe=2.8 at x=6 61

Figure 4-20(a): Pressure contour for: Area Ratio=2.0, Pi/Pe=2.8 62

Figure 4-20(b): Pressure contour for: Area Ratio=2.0, Pi/Pe=2.8at x=6 62

Figure 4-21: Centerline Mach distribution for 1-D and 3-D flow: Area Ratio=2.0, Pi/Pe=2.8 63

Figure 4-22: Mach contour for: P0=100, T0=3.5,Z=0 67

Figure 4-23: Pressure contour for: P0=100, T0=3.5,Z=0 67

Figure 4-24: Centerline Mach distribution for P0=100, T0=3.5 68

Figure 4-25(a): Mach contour for P0=10, T0=1.5,Z=0 69

Figure 4-25(b): Mach contour for P0=10, T0=1.5 at the exit section 69

Figure 4-26: Pressure contour for P0=10, T0=1.5,Z=0 70

Figure 4-27: Centerline Mach distribution for P0=10, T0=1.5 71

Figure 4-28: Mach contour for P0=25, T0=2.5,Z=0 72

Figure 4-29: Pressure contour for P0=25, T0=2.5,Z=0 72

Figure 4-30: Centerline Mach distribution for P0=25, T0=2.5 73

Figure 4-31: Mach contour for P0=50, T0=2.5,Z=0 74

Figure 4-32: Pressure contour for P0=50, T0=2.5,Z=0 74

Figure 4-33: Centerline Mach distribution for P0=50, T0=2.5 75

Figure 4-34(a): Mach contour for P0=80, T0=2.5,Z=0 76

Figure 4-34(b): Mach contour for P0=80, T0=2.5 at x=6 76

Figure 4-35(a): Pressure contour for P0=80, T0=2.5,Z=0 77

Figure 4-35(b): Pressure contour for P0=80, T0=2.5 at x=66 77

Figure 4-36(a): Mach contour for P0=150, T0=3.0,Z=0 at time t1 78

Figure 4-36(b): Mach contour for P0=150, T0=3.0,Z=0 at time t2>t1 78

Figure 4-36(c): Mach contour for P0=150, T0=3.0,Z=0 at time t3>t2 79

Figure 4-37: Variation of the inlet stagnation pressure with time 80

Figure 4-38(a): Time dependent problem; Mach contour at time t1 81

Figure 4-38(b): Time dependent problem; Mach contour at time t2>t1 81

Figure 4-38(c): Time dependent problem; Mach contour at time t3>t2 82

CHAPTER I

Introduction

Mathematical physics is the study of mathematical models that describe observed physical phenomena. Computational fluid dynamics is a branch of mathematical physics that deals with the numerical solutions of several mathematical models, explaining the physics of fluid flow. Many important problems, of frequent occurrence in the field of fluid mechanics and dynamics and, especially, in transonic and supersonic fluid flows, can be solved and analyzed by the Euler equations. The Euler equation models are not just the study of non-viscous fluids, but of fluids with such small values of viscosity, that tangential stresses are small compared to the normal pressure exerted by the fluids.

There are three different approaches for modeling convective terms in the discretized Euler equations: Artificial viscosity, Upwind flux difference splitting and Godunov-types schemes. First, we are going to have a look at a brief history of these models.

1-1 Artificial Viscosity

Consider a one-dimensional system of conservation laws

$$\frac{\partial u}{\partial t}+\frac{\partial f}{\partial x}=0 \qquad (1.a)$$

where u and f are column vectors. The above system can be written as a quasi-linear system

$$\frac{\partial u}{\partial t} + A(u)\frac{\partial u}{\partial x} = 0 \tag{1.b}$$

where A is the Jacobian matrix $\frac{\partial f}{\partial u}$. The numerical fluxes $f^*_{i+1/2}$ defined by

$$f^*_{i+1/2} = (f_i + f_{i+1})/2 - \frac{\Delta t}{2\Delta x} A_{i+1/2}(f_{i+1} - f_i) \tag{2.a}$$

Different versions of the non-linear Lax-Wendroff schemes [1] can be written according to Lax and Wendroff as

$$f^*_{i+1/2} = (f_i + f_{i+1})/2 - \frac{\Delta t}{2\Delta x} A_{i+1/2}(f_{i+1} - f_i) - D(u_i, u_{i+1}).(u_{i+1} - u_i) \tag{2.b}$$

where D is any possible function of $(u_{i+1} - u_i)$ which goes to zero at least linearly with $(u_{i+1} - u_i)$. The function D must have the dimension of A, that is the dimension of a velocity times density, and therefore $D\Delta x$ has the dimensions of viscosity if u represents a velocity component. Lax and Wendroff called D the artificial viscosity.

In order for $D_{i+1/2}$ to have a stabilizing influence, it has to be positive [1]. However, one can also define D as a polynomial function of $(u_{i+1} - u_i)$, which is often done in practical implementations of artificial viscosity terms.

Jameson and Turkel [1] are just two examples of many researchers who have investigated some external flows with artificial viscosity schemes. The forms of artificial viscosity terms are not arbitrary but any form of non-vanishing dissipation will be sufficient to implement the entropy condition and exclude expansion shocks as shown by Lax [see ref. 1].

Jameson applied a blend of the expressions of artificial viscosity, addition of third-order derivatives plus higher-order derivatives, considering shock-capturing properties [see ref. 1]. In this approach, the third derivative term is switched off, when the quantity of artificial viscosity dominates. The same formulation has also been applied by Pulliam [2].

Swanson and Turkel [3] modified the artificial dissipation (viscosity) model, including boundary treatment, for solving the Euler and Navier-Stokes equations, and then they used a central differencing algorithm to investigate various models. In that work, the artificial dissipation model introduced by Jameson, Schmidt and Turkel is reviewed.

Reddy and Jacocks[4] used a locally implicit method for solving the Euler equations with finite volume spatial discretization and Jameson-type artificial dissipation terms.

1-2 Upwind Schemes

The family of the upwind schemes may be taken back to Courant, Iscaacson and Reeves who introduced the physical properties of the flow equations into the discretized formulation, which led to a new