

IN THE NAME OF GOD

**Analysis and Shape Optimization
of
Frictional Contact Problems**

By

Sima Ziaee

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EVALUATED AND APPROVED BY THE THESIS COMMITTEE
AS : EXCELLENT

For
[Signature]..... G.Karami, Ph.D., Professor of
Mechanical Engineering (chairman)

[Signature]..... M.Eghtesad, Ph.D., Assistant Professor of
Mechanical Engineering

[Signature]..... M.Farid, Ph.D., Assistant Professor of
Mechanical Engineering

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Dedicated to

My dear family

f. 1995

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ABSTRACT

ANALYSIS AND SHAPE OPTIMIZATION OF FRICTIONAL CONTACT PROBLEMS

By
Sima Ziaee

Stress analysis of bodies in contact is an important class of structural mechanics problems. Contact stresses on the contact surface or somewhat beneath the surface would usually limit the load carrying capacity of the members in contact. In the last twenty years, considerable research has been focused on classical contact problems to develop computational techniques of determining the contact regions and their stress distributions, when the system configurations and applied loads are known. Due to unknown nature of contact regions, contact problems are classified as inverse problems. In a contact system, excessively high contact stress may occur over parts of the contact region and this greatly affects the performance, reliability and lifetime of the structures involved. Therefore inverse problems have received

considerable attention, whereby the contact system configuration is to be optimized to achieve the best possible distribution of stress over the contact regions.

In this work, Augmented Lagrangian Method (ALM) is used for studying general frictional elastic contact problems. A simple but efficient algorithm is proposed for the shape optimization with desired contact traction. To evaluate the accuracy and the capability of the two algorithms, several simple examples are solved. Results are compared with COSMUS/M and good agreements are obtained.

TABLE OF CONTENTS

Content	Page
LIST OF TABLES	X
LIST OF FIGURES	XI
CHAPTER 1 : THE CONTACT PROBLEM	
1.1. Introduction	1
1.2. Analytical Treatment of Contact	3
1.3. Numerical Treatment of Contact	4
1.4. Classification of Contact Problems	14
1.4.1. Advancing and receding contacts	14
1.4.2. Frictionless and frictional contacts	16
1.4.3. Static and dynamic contacts	19
1.4.4. Hertzian and non-Hertzian contacts	19
1.5. Interface Boundary Conditions	20
1.5.1. Separation mode	21
1.5.2. Contact mode	21
1.5.3. Stick mode	21
1.5.4. Coulomb slip mode	22

1.5.5. Slip frictionless mode	22
1.6. Load Incremental Method	22

CHAPTER 2: 3D ELEMENT FEM FORMULATION

2.1. Introduction	26
2.2. 3D Solid Element	27
2.3. Basic Equations	28
2.3.1. Equation of motion	28
2.3.2. Strain-displacement relations	28
2.3.3. Stress-Strain relations	29
2.3.4. Boundary conditions	29
2.4. Weak Formulations	30
2.4.1. Principle of virtual work in matrix form	30
2.5. Finite Element Model	31
2.6. 3D Solid Elements development	35
2.6.1. Constant strain tetrahedron	35
2.6.2. Linear strain tetrahedron	36
2.6.3. Trilinear hexahedron	36
2.6.4. Quadratic hexahedron	37
2.7. Stiffness Matrix	38
2.7.1. Tetrahedron Elements	38
2.7.2. Hexahedron Elements	41
2.8. Mesh Preparation	46

CHAPTER 3 : AUGMENTED LAGRANGIAN METHOD FOR CONTACT ANALYSIS

3.1. Introduction	50
3.2. History	52
3.3. Contact Mechanics Background	53
3.3.1. Governing equations	54
3.3.2. Contact conditions	54
3.3.3. Coulomb friction law	55
3.3.4. Weak form	56
3.3.5. Augmented Lagrangian formulation	57
3.3.6. Primary nested augmented Lagrangian algorithm for frictional contact	60
3.3.7 How does augmented Lagrangian work?	65
3.4. Contact discretization	67
3.5. The treatment of rigid body displacements	71
3.5.1. Coupling with additional springs	72
3.5.2. Considering certain nodes as fixed	75
3.6. Quasi-Newton solver	73

CHAPTER 4: CONTACT SHAPE OPTIMIZATION

4.1. Introduction	85
4.2. History	86

4.3. Evolutionary structural optimization (ESO)	88
4.4. Evolutionary procedure for contact problems	89
CHAPTER 5: RESULTS AND CONCLUSIONS	
5.1. Conclusions	143
5.2. Suggestions for future study	144
REFERENCES	145
ABSTRACT AND TITLE PAGE IN PERSIAN	

LIST OF TABLES

Table	Page
2.1: Five tetrahedra	48
2.2: Six tetrahedra	49
5.1: Comparison between theory and program	97

LIST OF FIGURES

Figure	Page
2.1: Constant strain (four -node) tetrahedron	47
2.2: Linear strain (ten-node) tetrahedron	47
2.3: Trilinear (eight-node) hexahedron	47
2.4: Quadratic (20- node) hexahedron	47
2.5: Cube for tetrahedral division	47
2.6: Division of a cube into five tetrahedra	48
2.7: Division of a cube into six tetrahedral	49
3.1: Notation for the frictional obstacle problem in small deformation	81
3.2: Typical interface	81
3.3: Simple one dimensional non-linear spring example. Schematic(a) and one dimensional idealization (b)	82
3.4: Graphical illustration of solution of the non-linear spring example, showing (a) non-linear response of spring as a function of length change, (b) application of the standard penalty method, with solution u^m for varying penalty parameters ϵ^m given by intersection of $-F^{int}$ with $F_c(\epsilon^m)$, and (c) demonstration of the augmented Lagrangian iteration process, where constraint satisfaction is improved while using a moderately sized penalty	83

3.5: Graphical representation of successive non-linear equation to be solved within the augmented Lagrangian framework. Iteration process finds zero of current R curve; multiplier update defines the next residual curve by shifting R in proportion to the gap g evaluated at the root	84
4.1.a: A Michell type structure.	93
4.1.b: Design domain for the Michell type structure	93
4.1.c: Evolutionary path towards the optimum Michell type structure.(Initial rejection ratio $RR_0=1\%$ evolution rate $ER=.5\%$. Fig. (j) gives $\theta=\pi/4$, which agrees with the analytical solution.)	94
5.1 a) Geometry b) Modeling of Example 1	97
5.2: Geometry (a): before (b): after optimization of example2	99
5.3: Geometry and loading of Example 3	101
5.4: Deformed shape of Example 3 near contact zone for $P=4000$ lb.	101
5.5 Normal pressure of Example 3 for $P=4000$ lb.	102
5.6: Normal pressure of Example 3 for $P=5000$ lb.	102
5.7: (a): Geometry and loading (b): Modeling of Example.4	104
5.8: Undeformed shape of Example 4 in x-y plane	105
5.9: Deformed shape of Example 4 in x-y plane	105
5.10: Normal pressure of Example 4 in contact zone	106
5.11: Geometry and loading of Example 5 (section a, b and c)	108
5.12: Normal pressure of Example 5.a for $P=3$ kN/cm	108
5.13: Normal pressure of Example 5.a for $P=4$ kN/cm	109
5.14: Normal pressure of Example 5.a for $P=7.5$ kN/cm	109

5.15: Normal pressure of Example 5.a for $P=10$ kN/cm	110
5.16: Normal pressure for frictional contact of Example 5.a. ($E=1000$ kN/cm ² , $P=150$ KN)	110
5.17: Tangential stress for frictional contact of Example 5.a ($E=1000$ kN/cm ² , $P=150$ kN)	111
5.18: Comparison between frictionless and frictional normal stress of Example 5.a ($E=1000$ kN/cm ² , $P=150$ kN)	111
5.19: Normal pressure of Example 5.b for $P=1.75$ kN/cm	112
5.20: Normal pressure of Example 5.b for $P=2$ kN/cm	113
5.21: Normal pressure of Example 5.c for $P=3$ kN/cm	114
5.22: Geometry and loading of Example 5.d	115
5.23: Frictionless normal pressure of Example 5.d	116
5.24: Frictional normal pressure of Example 5.d, $\mu=.6$	116
5.25: Tangential stress of Example 5.d, $\mu=.6$	117
5.26: Comparison between frictionless and frictional cases. of Example 5.d	117
5.27: Geometry and loading of Example 6	119
5.28: Deformed shape for frictionless case	119
5.29: Deformed shape for frictional case.	119
5.30: Normal pressure of Example 6 for frictionless case	120
5.31: Normal and tangential stresses of Example 6 for frictional case	120
5.32: Geometry and loading of a) Example 7.a b) Example 7.b	122
5.33: Undeformed shape of Example 7.a	123
5.34: Frictionless deformed shape of Example 7.a ($\mu=0$)	123

5.35: Frictional deformed shape of Example 7.a ($\mu=.1$)	124
5.36: Frictional deformed shape of Example 7.a ($\mu=.2$)	124
5.37: Normal pressure of Example 7.a for $\mu=0$	125
5.38: Normal and tangential stresses of Example 7.a for $\mu=0.1$	125
5.39: Normal and tangential stresses of Example 7.a for $\mu=0.2$	126
5.40: Undeformed shape for Example 7.b ($P = 4 \text{ kN/cm}^2$)	126
5.41: Deformed shape for Example 7.b	127
5.42: Normal and tangential loads for Example 7.b	127
5.43: Geometry and loading of Example 8	129
5.44: Udeformed shape of Example 8 in x-y plane	130
5.45: Deformed shape of Example 8.a-1(scale = 100)	130
5.46:Normal pressure of Example 8.a-1	130
5.47: Deformed shape of Example 8.a-2	131
5.48: Normal pressure of Example 8a-2	131
5.49: Deformed shape of Example 8.a-3	132
5.50: Normal pressure of Example 8a-3	132
5.51: Deformed shape for Example 8.b	133
5.52: Normal and tangential loads for Example 8.b	133
5.53: Comparison between frictionless and frictional case	133
5.54: Geometry and loading of (a): Example 9.a (b): Example 9.b	136
5.55: Comparison between normal pressure before and after optimization for Example 9.a	137
5.56: Designed gap space for Example 9.a	137
5.57: Normal pressure before optimization of Example 9.b	138

5.58: Desired normal force of Example 9.b	138
5.59: Designed gap space for Example 9.b	139
5.60: Comparison between desired and designed normal force	139
5.61: Stresses before optimization for Example 9.b	140
5.62: Desired normal force of Example.9b for frictional case	140
5.63: Designed gap space of Example 9.b	141
5.64: Comparison between desired and designed normal forces of Example 9.b	141
5.65: Desired normal pressure for Example 9.b	142
5.66: Comparison between designed gap space with and without friction for desired normal pressure Fig.5.65	142