Determination of stochastic volatility of Tehran Stock Market Return: Evaluation and Power of Alternative Tests

By

Leila Seyed Moosavi

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Determination of Stochastic Volatility of Tehran Stock Market Return: Evaluation and Power of Alternative Tests

Dr. Morteza Aalabaf Sabaghi

Dr. Abbas Shakeri

By

Leila Seyed Mousavi

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ALLAMEH TABATABAEI UNIVERSITY
DEPARTMENT OF
ACTUARIAL SCIENCE

The undersigned hereby certify that they have read and recommend to the Faculty of Graduate Studies for acceptance a thesis entitled "Applying Extreme Value Theory in Measuring Market Risks" by Leila Seyed Moosavi in partial fulfillment of the requirements for the degree of Master of Science.

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Supervisor: Dr. Morteza Aalabaf Sabaghi

Advisor: Dr. Abbas Shakeri

Examiner: Dr. Mehdi Taghavi

To

My husband

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Chapter 1

Introduction

1-1- Risk

The phenomenon of risk plays an important role in economic life. Without it, financial and capital markets would consist of the exchange of a single instrument each period, would reduce to that of accounting. A situation is said to involve risk if the randomness facing on economic agent can be expressed in terms of specific numerical probabilities.

The representation of individual's preferences over distributions of risk by the shape of their von Neumann-Morgenstren utility functions provides the first step in the modern economic characterization of risk. After all whatever the notion of riskier means, it is clear that bearing a random wealth $\overline{X}$ is riskier than receiving a certain payment of $\overline{X} = E(x)$ that is the expected value of the random variable $\overline{X}$. We therefore have that an individual would be risk averse that is always prefer a payment of $\overline{X} = E(X)$(and obtaining utility $E\left(U\left(\overline{X}\right)\right)$if and only if his utility function was concave.

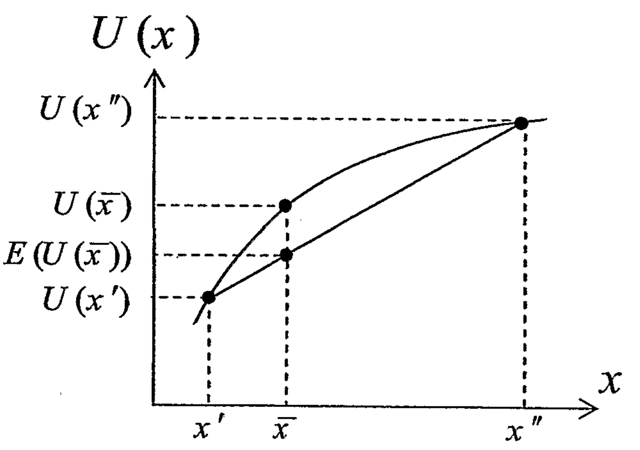

Figure (1-1): Utility function of a risk averse individual

Another type of individual is a risk lover. Such an individual would have a convex utility function, and would accordingly prefer receiving a random wealth $\overline{X}$ to receiving its mean $E\left(\overline{X}\right)$ with certainty.

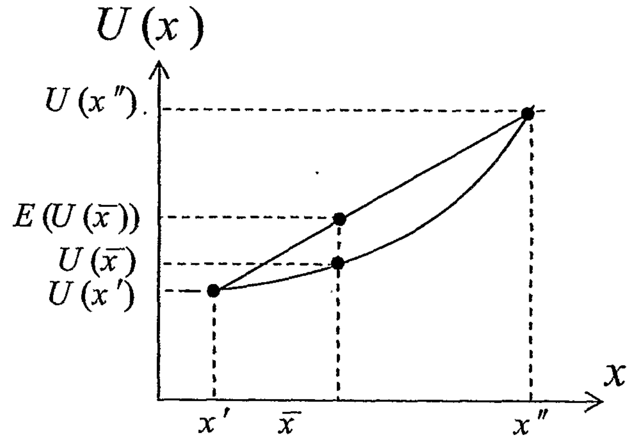

Figure (1-2): Utility function of a risk-loving individual

1-2- *Volatility*[1]

Consider the following two graphs

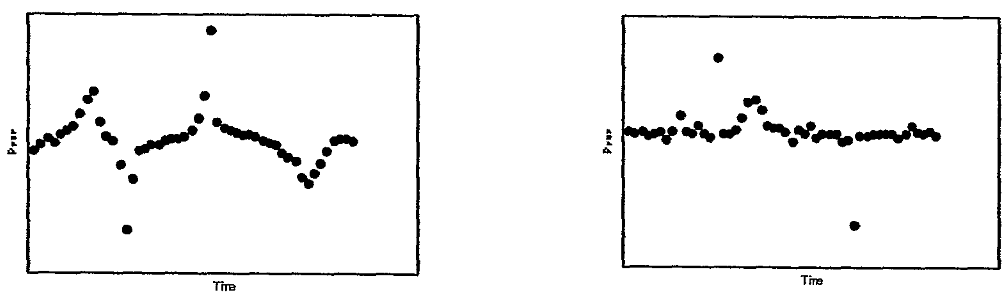

Figure (1-3): Examples of two time series of prices

They depict time series of prices for two hypothetical assets. We may think of the price series on the left as more risky. We say it is more "volatile" of the two. We formalize this by defining volatility as follows. Let $\ldots, Q_{t-2}, Q_{t-1}, Q_t, Q_{t+1}, \ldots$ be a stochastic process. Its term Q_t may represent price, accumulated value, exchange rates, interest rates, etc. The volatility of the process at time $t-1$ is defined as the standard deviation of the time t return. Typically, log returns are used, so the definition becomes:

$$\textbf{\textit{Volatility}} = std\left(\log\left(\frac{Q_t}{Q_{t-1}}\right)\right) \qquad (1\text{-}1)$$

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[1] See Appendix B. Section "volatility"

Where, log denotes a natural logarithm. However, simple returns are some time used. This is especially true in the context of portfolio theory.

If we assume that returns are conditionally homoskedastic[1], definition (1-1) is precise .However, if they are conditionally heteroskedastic (A condition where a stochastic process has non-constant second moments), we need to clarify the definition. Does volatility at time $t-1$ represents the unconditional standard deviation of the time t log return. Or does it represent the standard deviation of the time t log return conditional on information available at time $t-1$. The answer is the latter. To emphasize this, we might express definition (1) as:

$$\text{Volatility} = std\left(\log\left(\frac{Q_t}{Q_{t-1}}\right)\right)_{t-1} \quad (1\text{-}2)$$

Where, the subscript $t-1$ indicates that the standard deviation is conditional on information available at time $t-1$.

Another issue in defining volatility is that of the unit of time on which it is based. The standard deviation of a stock's price return over a day might be 0.01. Over a year it might be 0.16. Accordingly, for any quantity, we might speak of its daily volatility, weekly volatility, annual volatility, etc. All would be distinct notions. This leads to the question of whether, given a volatility based upon one time unit, is there a way to convert it to an equivalent volatility based upon another time unit. As a general rules the answer is no. To understand why, consider figure (1-4)

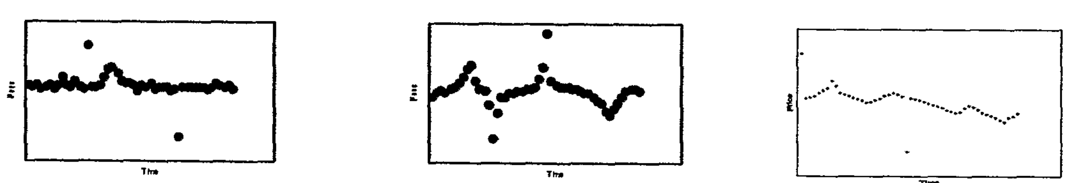

Figure (1-4): Examples of different time series

The first time series is a realization of a mean reverting process. There is about as much uncertainty in the price of a day in the future as there is a month in

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[1] In statistics, a sequence or a vector of random variables is homoskedastic if all random variables in the sequence or vector have the same finite variance. The complementary notion is heteroskedastic

the future. If the one day volatility is 0.02, then the monthly volatility might also be 0.02.

The second time series is a realization of a random walk. The price is more uncertain in a month in the future than in a day in the future. If the daily volatility is 0.02, then the monthly volatility would be 0.05.

The third time series is a realization of a process that tends to follow long term trends. Because of those trends, there is far more uncertainty in prices a months in the future than a day in the future. If the daily volatility is 0.02, the monthly volatility might be 0.08.

As this example illustrates, volatilities for different units of time are fundamentally different notions. There is no direct relationship between, say weekly volatility and an annual volatility. However, there is an exception to this observation. The exception is called the square root of time rule. If fluctuations in a stochastic process from one period to the next are independent (i.e. there are no serial correlations or other dependencies) volatility increases with the square root of the unit of time. Any price that follows a random walk, Brownian motion or Geometric Brownian motion satisfies this independence condition. The square root of time rule is exact if volatilities are based upon log returns. It is approximately correct if volatilities are based upon simple returns.

One approach to estimating volatilities is to apply techniques of time series analysis to historical data for the variable whose volatility is to be estimated. Volatilities calculated in this manner are called historical volatilities. These are routinely used in applications other than financial engineering, such as value-at-risk or portfolio theory, where volatilities are required for quantities on which options are not traded. They might also be used by financial engineers for underliers for which implied volatilities are unavailable.

1-3- Risk and Volatility

Financial risk has been traditionally associated with statistical uncertainty on the final outcomes. Its traditional measure is volatility. We will note by $R(T)$ the logarithmic return on the time interval T , defined by

$$R(T) = Log[\frac{X(T)}{X_0}] \qquad (1\text{-}3)$$

where $X(T)$ is the price of the asset X at time T, this definition is approximately equivalent to

$R(T) = [\frac{X(T)}{X_0}] - 1$. If $p(X,T \mid X_0,0)$ is the conditional probability of finding $X(T) = x$ within dx, the volatility σ of the investment is the standard deviation of $R(T)$, defined by:

$$\sigma^2 = \frac{1}{T}\left[\int P(X,T \mid X_0,0) R^2(T) dx - \left(\int P(X,T \mid X_0,0) R(T) dx\right)^2 \right] \qquad (1\text{-}4)$$

The volatility is in general chosen as an adequate measure of risk associated to a given investment.

1-4- Security

A derivative is a security that has a pay off that depend on one or more risky assets. A risky asset has value following a random process such as those described above. For example a stock is risky asset with the price by a combination of the dividend yield and dividend index series. Other examples are option or bond, etc.

A common approach in the modeling of financial assets is to assume that the proportional price of an asset form a Gaussian process with stationary independent increments. The celebrated Black-Scholes option pricing formula is based on such a premise. The success and longevity of the Gaussian modeling approach depends on two main factors: firstly the mathematical tractability of the model, and secondly the fact that in many circumstances the model provides a reasonable and simple approximation to observed market behavior.

An immediate corollary of the Gaussian assumption is that behavior of the asset price can be summarized by two parameters, namely the mean and the standard deviation of the Gaussian variables. In finance the standard deviation is renamed the volatility. Volatility is a key concept because it is a measure of uncertainty about future price movement, because it is directly related to the risk associated with holding financial securities and hence affects consumption /

investment decisions and portfolio choice, and because volatility is the key parameter in the pricing of stocks, options, and other derivative securities.

Stock market indexes reflect value change in stocks they represent and play the rule of benchmark in evaluating the performance of investment mangers. They are also used as the underlying value of stock index futures and stock options.

In the original Black-Scholes model, the risk is quantified by constant volatility parameter. A natural generalization is to model the volatility by a stochastic process. In reality the volatility process can not be directly observed.

Stochastic volatility models have become popular for derivative pricing and hedging in the last ten years as the existence of a non flat implied volatility surface has been noticed. This phenomenon stands in contradiction to the consistent use of a classical Black-Scholes (constant volatility) approach to pricing options and stocks and similar securities. However, it is clearly desirable to maintain as many of the features as possible that have contributed to this model's popularity and longevity, and the natural extension pursued in the literature and dynamics of the underlying asset price model.

In a stochastic volatility model the volatility is changing randomly according to some stochastic differential equation or some discrete random processes. A stochastic volatility model introduces more random sources than traded assets. According to general market theory the model is not complete since the number of random sources is greater than the number of underlying traded assets.

However stock price returns that are estimated shows that volatility fluctuates randomly around a mean level. The process is said to be mean-reverting.

1-5- Aims

We will present in this thesis the theory underlying most of these models, the theory of Random Walks. Then we will discuss different hypothesis that has been proposed, in the Bachelier-Osbrone(1959) models concerning standard deviation. This parameter is crucial since it represents the volatility of the market. After having tested these models with empirical data, we will see that the structure of the market implies neither a constant standard deviation, as thought initially, nor an infinite standard deviation:

The Probability Density Function (PDF) of the log – returns exhibits fat tails[1] and kurtosis, with finite standard deviation.

Then we will focuses on one of these models, published by Dragulesco and Yakavenko(2002). Their paper introduces a new model for volatility of stock market indexes. The proposed probability density function of stock returns seems to fit empirical data much better than previous models. We will double-check their results and propose a methodology test their model against empirical data.

1-6- Quantitative Methods

The multiplicative diffusion process known as the geometric Brownian motion (GBM) has been widely accepted as on of the most universal models for speculative market. Bachelier (1900) used an ordinary random walk to explain stocks prices in Paris bourse. This model was refined in Osborne (1959). Theory of Random Walks, claims that successive price changes ($P_\tau - P_0$) or price returns ($\frac{P_\tau}{P_0}$) are independent, identically distributed random variables. This i.i.d. hypothesis has been studied by Fama (1965) and is still discussed today.

The assumption of i.i.d price returns is common among many models. This is so called Bachelier-Osborne (1959) model and states that price returns have a constant finite volatility over a given period of time ("time lag τ") e.g. one day, one week, One month, etc. This theory results in a log –normal distribution for price returns and volatility proportional to the square root of time lag, i.e. the weekly volatility will be about $\sqrt{5}$ times higher than daily changes. But it is now known that price returns do not follow a Gaussian distribution[2], since they exhibit kurtosis and fat tails. Hence, models have introduced volatility. Mandelbort (1963) first introduced volatility into his model to tackle fat tails which lead to stable Pareto-Levy distribution of returns. Unfortunately, the hypothesis of infinite volatility supposes that the variance increases indefinitely with sample size, which is not verified by empirical data. Variance first increases then reach an upper bound.

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[1] See appendix B. Section "Fat tail"

[2] See appendix B. Section "Gaussian distribution"

For a long time, practitioners have tried to model and predict financial markets. With rapid growth of statistics and stochastic calculus since fifty years ago, new quantitative methods were born that seems to be able to handle the complexity of stock markets.

However, we show that the Bachelier-Osborne (1959) model fails to tackle high kurtosis and fat tails. Draguleseu and Yakovenk (2002) published an improvement of this model. We use the Dow Jones index and compare it with data from Tehran Stock Market.

The literature on this topic is numerous and we only mention a small sample of related works. This includes among others, Hull and White (1978), Cox-Ingersoll and Ross (1985), Cox-Ross and Rubinstain (1979),Heston (1993), Derman (1999), Britten- Jones and Neruberger (2000),Anderson and Andreasen (2000) and Aalabaf Sabaghi (2002).

In most cases these researchers consider option prices and valuation of options. However, in Iran no options are traded in open markets and there are no option prices to consider. But it is conceivable to find examples of options in some grey or in agricultural markets.

1-7- Dow- Jones Industrial Average (DJIA)

The earliest US stock market index goes back to 1884 when it was first compiled from only eleven stocks. Currently this index is based since 1928 starting at 100. The index is a simple price average based on thirty shares of the most important quoted companies on the New York Stock Exchange. It is now largely superseded by wider indices such as the S&P500 but it is useful for historical interests. It has chronicled the vagaries of the stock market, reaching its low point of 41 on 2 July, 1932, following the great crash, but currently stands in the mid-thousand range.

1-8- Tehran Stock Market (TSM)

Tehran Stock Exchange Market was established in 1967. It has started its activity with some transaction on the stocks of Industrial and Mining Development Bank of Iran. After that Pars Oil Association, government bonds, Treasury bonds, bonds of Industrial Proprietorship Extension Organization and

Abbas Abad bonds came to Tehran Stock Market. Since many trade firms received tax exemption status under Iranian tax laws, the TSM has been an improvement in trading activity. During 11 years of activity before Islamic revolution of Iran the number of organizations, banks and insurance companies from 6 institutions with 6.2 billion rials total assets in 1967 reaches to 105 institutions with 220 billion rials of assets in 1979.

The first five year development plan after the Islamic revolution in 1979 brought many changes to the Tehran Stock Exchange.

First of all was ratification of law of banks direction bill in 1979 that commercial and professional banks combined and became national. After that private insurance companies combined and became government owned companies.

Also ratification of law of protection and extension of Iran's industries in 1979 led to exiting many of economical institutions from TSM, so their number fell from 105 in 1979 to 56 in 1989. During those years TSM had its interval period. Since 1990 and through the first five year development plan, there has been a considerable increase in TSM activity. These activities have fuelled initiatives for privatizations.

So politicians want to transfer some of government's duties to private part for collecting dispersed assets sources and conduct them to contribute in economical activities. Because of that decision the number of institutions reaches to 325 in 2007.

Accepted firms (manufactures and investment companies) in TSM are divided into two groups: producing firms that are 248 and investment firms that are 19.

1-9-Tobin q theory about investment

Investment in nature has some delays, modifies and modification costs, so we should attention this limits for choosing the time and amount of investment. In project assessment, calculating the present value of projects, and calculation the final efficiency of investment, most of prices such as products price and saved price are related to future and investors don't have exact knowledge of them. So, they should form their expectations in a reliable way about them. Anyway, unsurely about these variables (prices) are related to probability distribution form of probabilities amounts and their variances lead to adventure and risk cost.

James Tobin from Yale University presents a theory that implicitly has attention to delays, modifies and modification costs. This theory is called Tobin q theory. He introduce an scale in subject of q by using with market value of institutions stocks, machinery purchase cost and invest replacing costs; and say if q be bigger than 1, excess investment is profitable and should do. If q be equal to 1, it show investment balance and finally if q be less than 1, it means investment should reduce.

Tobin q can be introduced as

$$q = \frac{MV}{p^K} \qquad (1\text{-}5)$$

where MV is market value of an unit institution invest

p^K is price of that invest in market

Here we can introduce q as the ratio of market value of a unit invests to that replacing cost. Institution invest has a replacing cost that is equal to the price of au unit invest from invest goods market, but when this invest goods purchase by an investors and combine with his management and job organization and his produce, get a value that is completely differ from replacing cost and its primary market value. We know that, market forces in market is very powerful that can assess technical, managerial and competition condition of institution and its future economical view; and with attention to delays, modifies and expectations and risk, determine the stock price or price of institution invest in market. Certainly investors continuo to investment until that each unit of invest that he purchase in invest goods market with price of p^K and use in his institution, has more price than p^K in stock market; and investment balance point of institution is where that price of a unit invest be equal to that price in invest goods market. It means that $q = 1$. If q be greater than 1, means each invest unit that investor buys from invest goods market with of p^K and use them in frame of his management, get more value. So it is profitable for his to invest or q be equal to 1.

Now we can introduce q in other way:

$$q = \frac{J}{R^K} = \frac{J}{\left(r + d - \pi^K\right)p^K} \qquad (1\text{-}6)$$

Where J is pure output of each unit of invest

r is nominal interest rate

d is discount rate

R^K is the cost of use of a unit invest

Here q is the ratio of a unit of invest to that using cost. $q > 1$ Means pure outputs of each unit of invest in investment is greater than its using cost. So, it is profitable for investor to continuo to his investment. Therefore, investment is profitable until final use of a unit invest be equal to its using cost.

Relation of 1 &2 are equal, because in stock market, stock value of a unit invest is

$$MV = \frac{J}{\left(1 + r + d - \pi^K\right)} + \frac{J}{\left(1 + r + d - \pi^K\right)^2} + \ldots + \frac{J}{\left(1 + r + d - \pi^K\right)^n} = \frac{J}{\left(r + d - \pi^K\right)}$$

Because future output should discount with real output rate in financial market and friction the invest unit in each period. If we divide each side of this relation to p^K, will have

$$\frac{MV}{p^K} = \frac{J}{p^K\left(r + d - \pi^K\right)} \qquad (1\text{-}7)$$

We know that $p^K\left(r + d - \pi^K\right)$ is equal to R^K so,

$$\frac{MV}{p^K} = \frac{J}{R^K} = q \qquad (1\text{-}8)$$

If $q = 1$, it means that:

$$\frac{J}{R^K} = \frac{J}{p^K\left(r + d - \pi^K\right)} = \frac{J}{MV\left(r + d - \pi^K\right)} = 1 \qquad (1\text{-}9)$$

So

$$\frac{J}{MV} = \left(r + d - \pi^K\right) \qquad (1\text{-}10)$$

That is in balance point of investment, the ratio of stock output to its market value (percentage of output of each money unit in investment) is equal to real output rate in invest market by calculation of amortization. It means that investment output is equal to output of bonds with interest rate of r. Of course, reliance ability to this basis depends on the market value of institution stock invest determine on economical basis and real function of institution, and prices trend doesn't get bubble nature. In other way this basis misses its subject.

Tobin q that itself has emphasis on it to determine the balance investment level, give a useful way for investment formulizing. Because its components can be measure and calculate easily. Data of stock price are usually available from stock market and indexes of each year as components of national calculations. So we can say Tobin q present a good scale from investment position in economics that have profound and competition stock market. For example, in 1983 that real interest rate has been measured and rental price of invest has been showed unsuitable investment conditions, in fact investment was successful and q was greater than 1.

As we said if economic has competition markets, small basis of invest economic, that is the foundation of Tobin q theory, aren't very different from using cost of invest logic.

Villiam Branson also has worked out Tobin q from optimization condition of institution to determine desired investment level. He worked out the first degree condition to determine the desired investment level function of investment level via mid level optimization of present level of function of investment project.

$$P_t \frac{\partial Y}{\partial K} = d.P_t^I + rP_{t-1}^I - \left(P_t^I - P_{t-1}^I\right) \qquad (1\text{-}11)$$

Where P_t is price of product

$\frac{\partial Y}{\partial K}$ is final production of invest

d is discount rate

P_t^I is price of invest goods in period of t for investment

r is nominal interest rate

P_t^{I-1} is price of invest goods in period of $t-1$

The term in right side is cost of use from a unit invest and follow relation says that it should do invest and increase invest until final product value of invest be equal to its cost.

Branson rewrite above balance relation in this way:

$$P_t \frac{\partial Y}{\partial K} + (1-d)P_t^I = (1+r)P_{t-1}^I$$

$$\frac{\left(\frac{1}{r+1}\right)\left[P_t \frac{\partial Y}{\partial K} + (1-d)P_t^I\right]}{P_{t-1}^I} = 1 \qquad (1\text{-}12)$$

He define the left side of above relation as Tobin q, because it has been obtained from balance conditions. Here that is 1. left side numerator is present value equation of final invest product value and remained value of invest unit in future 12 months. In other term, present value is total of output and invest value in future year; and its denominator is price of invest goods or replacing cost at the end of previous period. If present value of remain value of invest unit and its gain is more than replacing cost $(q > 1)$, investment is profitable and should be continuo; and when $q < 1$, investment is not profitable; and balance of investment is where the present value with replacing cost is equal to 1.

There is a point: this q that here we calculate, is final q and has been get from first degree conditions and final equality in optimization process. But the q that has been get from dividing the market value to market price of a unit invest, is median q. Median q can be obtained from stock market information and index of invest goods prices in national calculations, but calculating of final q is difficult. However it has been said that if institutions perform in output position toward fix scale, final and median q will be equal.

An important point that we should attend, is that when for calculating of q stocks and market value of institutions is used, this market observed value has expectations that are related to all future periods in it and modifies toward risk and lack of confidence.

Of course it has been said the problem of final q and median q remain in some cases and these two scales give different results. For example, suppose that government take tax on some industries to prevent from publishing the pollutions. In this way according to median q, average value of institutions stocks decreases, but final q relate to investment in invest goods with less pollution and more