**The University of Sistan & Baluchestan**
**Graduate School**

**The Dissertation of Ph.D. in Mechanical engineering**

**Title:**

# Inverse analysis of radiation heat transfer in participating medium with variable refractive index

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**Feb. 2010**

**To my wife and my parents**

**Acknowledgments**

I would like to express my deep gratitude to my thesis supervisor, Professor Hosseini Sarvari for his persistent support, inspiration and outstanding guidance. He has given me constant encouragements during this project. I am also deeply indebted to Professor Mansouri for devoting his valuable time to improve this research. I would like to thank Professor Behzadmehr for his support and wish to express my sincere gratitude and thanks to Professor Lemonnier for offer in me a six month sabbatical training in France. His valuable guidance together with Professor Le Dez are appreciated.

For my friend Dr. Seidi, Dr. Sarhadi, Dr. Abedini and Dr. Razaghi I offer my most sincere words of gratitude.

Finally, very special thanks go to my wife, my Parents, my brother and my sisters for their supports.

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**Abstract**

In many engineering and practical problems, the medium has a variable refractive index such as heating of glass, thermal protecting coating, manufacturing of waveguide materials, optical measurement of flam, thermal barrier coating, elctrochoromic windows, etc. The variation of refractive index may be a reason of the structure or thermal effect caused by spatial and temporal variations. Therefore, retrieving of temperature and refractive index distributions, that play important role in radiative transfer in graded index medium, is very practical. In the present work an inverse analysis of 1-D absorbing, emitting and scattering graded index medium is performed to determine the temperature and refractive index distribution. In the first part, a serious attention is devoted to the direct problem, since solving the direct problem is a basic step in all inverse algorithms. The conservative and non-conservative form of radiative transfer equation of graded index medium in general orthogonal curvilinear coordinate system are presented, which has not been done till now. This formulation is simplified for 1-D case and the constant quadrature discrete ordinate method is used to solve it. The advantageous of presented method is its ability to model arbitrary variation of refractive index, while the previous similar method can only model the monotonic variation of refractive index. In the second part, three different inverse problems are solved. First, the source term (temperature distribution) is estimated through the knowledge of exit intensities at boundary surface by the conjugate gradient method. Estimation of refractive index distribution in a graded index medium by inverse methods through measured radiative heat transfer parameters is completely a novel idea and has not been done yet. Hence, in the second problem, the simultaneous estimation of source term and linear refractive index distribution is done through the knowledge of exit intensities at boundary surfaces by the conjugate gradient method and a two dimensional searching network approach. In the last case the arbitrary distribution of refractive index distribution is retrieved by a combination of the conjugate gradient method and Levenberg-Marquardt method. The measured data is radiative intensities at boundary surfaces and radiative heat fluxes inside the medium.

**Keywords:** Graded index medium, inverse analysis, curvilinear coordinate system

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**Nomenclatures**

| | |
|---|---|
| **a** | Vector of unknown coefficients |
| $c$ | Light velocity |
| **d** | Direction of descent |
| $E$ | Error |
| $g$ | Asymmetric scattering parameter |
| $G$ | Minimum of objective function at each discrete point of network |
| $h$ | Metric coefficient of coordinate system |
| **H** | Global rotation of local coordinate system |
| $F$ | objective function |
| $I$ | radiation intensity |
| $I_b$ | Blach body radiative intensity |
| **J** | Sensitivity matrix |
| $L$ | Optical path |
| $M$ | Number of angular elements |
| $M'$ | Number of discrete point of each search parameter |
| $N$ | Number of spatial elements |
| $n$ | Refractive index |
| $P$ | Order of source polynomial |
| $q$ | Radiative heat flux |
| **r** | Position vector |
| $S(\tau)$ | Source term |
| $s$ | Curvilinear abscissa along a path |
| **s** | Tangential unit vector to the path |

| | |
|---|---|
| $T$ | Temperature |
| $w$ | Angular weight |
| $Y$ | exit intensities on the surface boundaries |
| $\beta$ | search step size |
| $\varepsilon$ | Emissivity |
| $\zeta$ | random number |
| $\eta$ | Error in measurements |
| $\eta^*$ | Error in system parameters |
| $\rho$ | Conjugation coefficient |
| $\boldsymbol{\rho}$ | Charectrizes the rotation of local coordinate system |
| $\xi$ | Small positive number |
| $\kappa$ | Extinction coefficient |
| $\kappa_a$ | Absorption coefficient |
| $\kappa_s$ | Scattering coefficient |
| $\mu$ | Direction cosine |
| $\sigma_m$ | Standard deviation of measurement errors |
| $\sigma$ | Stephan-Boltzman constant |
| $\tau$ | Optical thickness |
| $\Phi$ | Scattering phase function |
| $\omega$ | Single scattering albedo |
| $\Theta$ | Represent a system parameter |

Subscript

| | |
|---|---|
| $i$ | Center values of spatial slice |

| | |
|---|---|
| $i \pm 1/2$ | Edge values of spatial slice with $i$ center |
| $k$ | Iteration number |
| $m$ | Center value of angular segment |
| 0 | Boundary at $\tau = 0$ |
| $L$ | Boundary at $\tau = \tau_L$ |
| $rel$ | Relative |
| $rms$ | Root mean square |

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# **Chapter 1.** Introduction

## 1.1. Introduction

Inverse problems are the problems that consist of finding an unknown property of an object, or a medium, from the observation of a response of this object, or medium, to a probing signal [1]. In the other words, in the direct problem the caused are given, the effect is determined; whereas in the inverse problem the effect is given, the cause (or causes) is determined. Thus, the theory of inverse problems yields a theoretical basis for remote sensing and non-destructive evaluation [1]. On mathematical physics view point, the aim of solving a direct problem is to find the solution of the partial differential equation with proper boundary and/or initial conditions. However, in an inverse problem governing differential equation is not completely defined or some of the boundary conditions or initial conditions are not specified, but some additional information is available. Through this information, the unknown conditions and parameters should be determined. [2]. Inverse analysis have a lot of practical and theoretical usage in all branches of science and engineering such as, physics, geophysics, hydrology, mathematics, astronomy, heat transfer and other disciplines. Inverse heat transfer problems are very important and practical. For example measurement of temperature in a furnace is a challenging problem. Due to the high temperature, the traditional thermometer is useless and we have to use more advanced methods. One possibility is to use ultrasound. The high temperature renders the gases in the furnace turbulent, thus changing their acoustic properties which in turn are reflected in the acoustic echoes. Now the forward model consists of the challenging problem of describing the turbulence as a function of temperature plus acoustic wave propagation in the medium, and its even more challenging inverse counterpart of determining the temperature from acoustic observations [3]. Also, aerodynamic heating of space vehicles is so high during reentry in the atmosphere that the surface temperature of the thermal shield cannot be measured directly with temperature sensors. Therefore, temperature sensors are placed beneath the hot surface of the shield and the surface temperature is recovered by inverse analysis [4].

## 1.2. Difficulties in Solving Inverse Heat Transfer Problem

Inverse heat transfer problems are classified as ill-posed problems in a mathematical sense, because their solution may become unstable as the measurements contain error. The solution of a problem should be satisfied the following three conditions to classify as a well-posed problem [4].

1. The solution must exist
2. The solution must be unique
3. The solution must be stable under small changes to the input

Compared to the well posed problem, the inverse problem may have no solution, or have multiple solutions. The main difficulty is that the solution may not be stable if there is error in input data. So we need special methods and algorithms to stabilize the solution. But, these methods do not guarantee the correct solution. Even if we get a stable solution for the inverse problem, it may not be acceptable as it is not the solution or the problem yields multiple solutions. So care must be taken during solving an inverse problem and the answers should always be looked as suspicious.

## 1.3. Classification of Inverse Problems

There are several classifications of inverse heat transfer problems related to the methods, usage, etc. Some of these classes are brought briefly in following lines:
Inverse problem can be solved as parameters estimation or as function estimation approach. In parameters estimation approach a finite number of parameters are to be determined. These parameters can be constant thermophysical properties such as thermal conductivity or absorption coefficient of medium. Also if some information is available on the functional form of the unknown quantity, the inverse problem is reduced to the estimation of few unknown parameters. If such information is not available, inverse problem become function estimation in an infinite dimensional space of functions.
Inverse heat transfer can be also classified in accordance with the mode of heat transfer process, such as [4]

1. Inverse heat transfer of conduction
2. inverse heat transfer of convection
3. inverse heat transfer of surface radiation
4. inverse heat transfer in participating medium
5. inverse heat transfer of simultaneous conduction and radiation
6. inverse heat transfer of simultaneous conduction and convection
7. inverse heat transfer of phase change

Another classification can be one based on the type of causal characteristic to be estimated. For example:

1. Inverse heat transfer of boundary condition
2. Inverse heat transfer of thermophysical properties
3. Inverse heat transfer of initial condition
4. Inverse heat transfer of source term
5. Inverse heat transfer of geometric characteristics of a heated body

Inverse heat transfer problems can be one, two or three dimensional. Also they may be linear or nonlinear.

## 1.4. Inverse Radiative Heat Transfer and Graded Index Medium

Radiative heat transfer is important when the temperature of medium and/or boundaries is high. In these cases this mode of heat transfer is almost dominant to other modes of heat transfer. Also radiative heat transfer is the only mode of heat transfer that does not need material medium and can transfer across the vacuum (as the sun heats the earth). The medium is called participating if it affects the intensity rays that travel through it. The effects can be classified into two groups. One attenuates the intensities pass the medium, and the other has augmentation effects. Attenuation is due to absorption and out-scattering and augmentation is a result of emission and in-scattering. It is obvious that the radaitive heat transfer in a medium depends on the properties of the medium, such as absorption coefficient, scattering coefficient, refractive index and so on. One of the most important properties is the refractive index which is the ratio of the velocity of light in the vacuum to its value in the medium. In a medium with constant refractive index the intensity rays travel along a straight path, while in a medium with variable refractive index the rays travel along a curve path. In this case, if the medium experiences a continuous variation of refractive index, it is called graded index medium. The early investigations in participating media are restricted to constant refractive index. However, in many engineering and practical problems, the medium has a variable refractive index such as heating of glass, thermal protecting coating, manufacturing of waveguide materials, ray transporting through atmosphere, optical measurement of flame [5], thermal barrier coating, connectors, ,electrochoromic displays, sensors, bobbins circuit breaker, batteries, elctrochoromic windows, etc. [6]. The variation of refractive index may be as a result of the structure or thermal effect caused by spatial and temporal variations.

Refractive index and its variations depend on structure, chemical composition, thermal treatment and conditions, etc. Mass density, molecular polarizability and molecular weight are some parameters that affect the refractive index. For pure materials, the relation between these four quantities has been presented by Lorenz-Lorenz relation. [7]. Among these parameters, the effect of mass density on refractive index is dominant. For mass density<<1 the refractive index varies linearly with mass density [7]. The manufacturing process also affects refractive index. Photo-thermo-refractive (PTR) glass is an optical material that is a candidate for hologram writing. After UV-exposure and thermal treatment, local refractive changes are seen in PTR glass. This change can be a reason of local chemical changes and local residual stresses [8]. A study was down by Lumeau et al [8] shows that among these parameters, residual stresses are the main reason of local decrease of refractive index. [8]. Chemical composition can be another reason of the variation of refractive index. Aeropolymer are porous materials with low refractive index of 1.2-1.3. Polymide has high refractive index of 1.66. These materials do not have adjustable refractive index over wide range. Xerogels is a poor material that its refractive index can be adjusted by controlling the pore fraction or embedded micro or nano particles. Lisinki et al. [9,10] showed that mixed silica-titania xerogels and pure titania xerogels have tunable refractive index over wide range of 1.2-2.1.

Inverse analyses in radiative transfer are very important and have a lot of practical applications, such as remote sensing of atmosphere properties, the prediction of the temperature profile in a furnace or atmosphere or flame. As already mentioned, the radiative heat transfer in a semitransparent graded index medium has a lot of practical applications. Therefore, the inverse analysys of such media are also practical and important. First, we discussed about the direct problem of radiative transfer in graded index medium, since solving direct problem is a part of all inverse algorithms. Then we discuss the inverse problem.

### 1.4.1. Direct Problem of Radiative Transfer in Graded Index Medium

The first idea of solving radiative transfer equation (RTE) in graded index medium was to divide the medium to layers and in each layer the refractive index has a constant value. Such method can be found in the works of Siegle and Spuckler [11, 12] By the same idea of dividing the layer with variable refractive index to sub layers with constant refractive index, Xia et al. [13] analyzed the thermal emission and volumetric absorption in a graded index semitransparent medium. More complicated situation of steady and transient coupled radiative-conductive heat transfer in a graded index slab was studied by Yi et al. [14].
Curved ray tracing technique that was developed by Ben Abdolah and Dez [15-19] is another method of modeling radiative transfer in graded index medium. This technique was modified and extended by other investigators. Huange et al. [20] using a pseudo-source adding method combined with curved ray tracing technique and obtained the temperature field inside an absorbing-emitting graded index semi-transparent slab with diffuse gray walls in the radiative equilibrium. In this work the variation of refractive index was assumed to be linear. This combination was to deduce the radiative intensities on the gray walls. They also extended their method to account the arbitrary variation of refractive index by discretization of medium and assumption of local linear approximation for refractive index distribution. Two kinds of sinusoidal variation of refractive index were considered as the case studies [5]. Xia et al. [21] obtained the non-dimensional radiative flux and temperature distribution in a semitransparent absorbing, emitting graded index slab. The modes of heat transfer were radiation