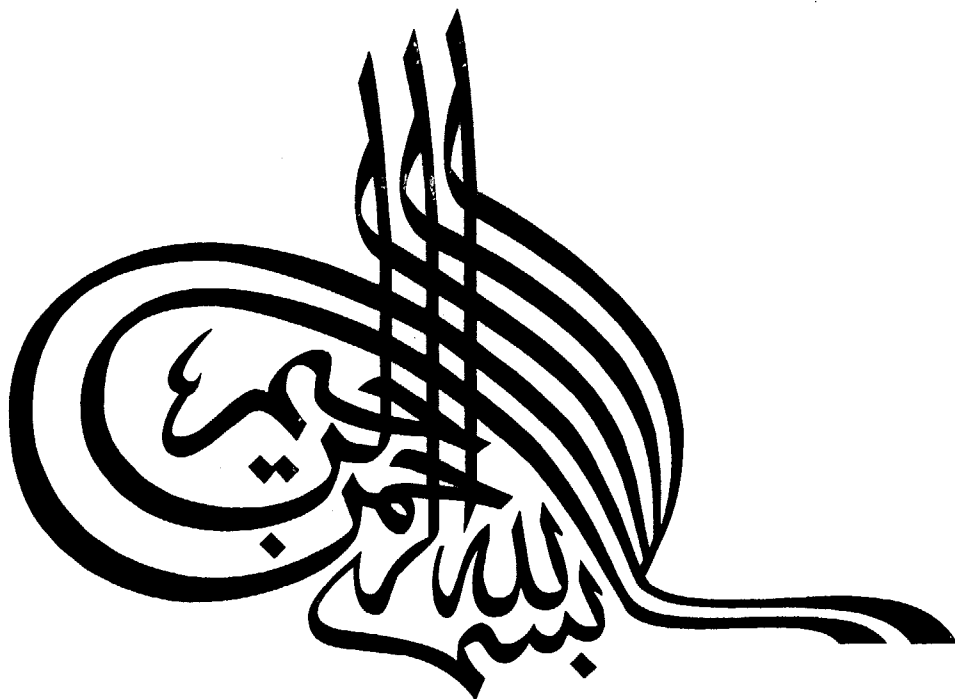


مرکز اطلاعات و مدارک علمی ایران  
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**IN THE NAME OF GOD**  
**PHOTONIC COLLAPSE AND REVIVAL**  
**IN**  
**THREE-LEVEL ATOMS**

**BY**

**ALI FALLAH ZADEH**

**THESIS**

**SUBMITTED TO THE SCHOOL OF GRADUATE STUDIES IN**  
**PARTIAL FULFILMENT OF THE REQUIREMENTS FOR**  
**THE DEGREE OF MASTER OF SCIENCE (M.Sc)**

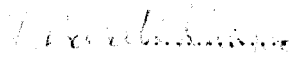
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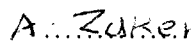
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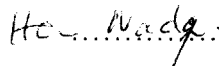
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MAY 2001



## ACKNOWLEDGMENT

I would like to express my thanks to Dr.M.M.Golshan for his supervision during the preparation of this work. I also wish to thank Dr.M.Barati, Dr.A.N.Zakeri and Dr.H.Nadgaran who read the manuscript and made helpful suggestions.

# ABSTRACT

## PHOTONIC COLLAPSE AND REVIVAL IN THREE-LEVEL ATOMS

BY  
ALI FALLAH ZADEH

In this thesis, the phenomenon of photonic collapse and revival in the interaction of three-level atoms ( $\Lambda$ -configuration) with two electromagnetic fields is studied. We investigate this interaction firstly when, one of the fields is quantum mechanically coherent and the other is classical, secondly, when one of the fields is quantum mechanically coherent one and the other is regular and lastly, when both fields are quantum mechanical and coherent. For each of the aforementioned cases, the atoms are initially assumed to be in their ground states or in a maximal atomic coherency. For each case, the mean photon numbers and atomic level populations are calculated. We then plot these quantities, as functions of time, for different values of atom-light interaction parameters. From these plots the following more important results may be deduced. (1) In all cases the phenomenon of collapse and revival in the photonic mean number and atomic level populations occur. (2) As the atomic system is initially in a coherent superposition of the two lower states, the mean photon number, while depending on the relative phase of the induced transition dipole moments, is enhanced. The atomic level populations would also depend upon this relative phase. (3) The

regular quantum field, as well as the coherent one, experiences the phenomenon of collapse and revival, showing a coherent behavior. In addition to detailed mathematical calculations, the physical reasons for such occurrences are also given.

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# CHAPTER 1

## INTRODUCTION

The interaction of light and matter, from the early stages in the history of man-kind to modern days, has generated much interest and influence. Such is, perhaps, due to the fact that generation, detection and analysis of the electromagnetic (EM) interaction may be done in relatively simple and settled ways. In fact, one may assert that basically all modern developments (for example, quantum optics and lasers, etc.) rely on the interaction of *electromagnetic* fields and matter. It is the main aim of this thesis to investigate the interaction of three-level atoms, as a model of physical reality, with *quantum mechanical* EM fields.

The interaction of EM fields and atoms is usually formulated in term of two active atomic levels and a single-mode EM field. This model, known as the Jaynes-Cummings Model [1,2], is rather simple and has served to investigate vast aspects of light-matter interaction [3,4]. An important result of such investigations, with direct practical applications [5], is the phenomenon of photonic *collapse* and *revival* [6]. The phenomenon of photonic collapse and revival occurs when the interacting EM field is initially in its *coherent state*, being a particular superposition of single-mode photonic number states [7]. In such phenomenon the mean

photon number periodically revives and collapses [8].

The Jaynes-Cummings Model may be advanced towards more physical reality by considering the matter, upon which the EM field acts, as a collection of three-level atoms [9-12]. The interaction of such a model with different sources of EM fields leads to some extraordinary results that would be absent in the two-level models. For example, if a three-level atom is acted upon by two classical (high intensity) EM fields, with suitable choices of amplitudes and polarizations, one may manage the atomic level populations to any desired values, thus *producing atomic coherency* [13]. The effect of these two fields is such that the produced atomic coherency remains intact indefinitely [14]. Another important consequence of considering the model of three-level atoms is *lasing without population inversion* [15]. To this end, one uses *a classical and two quantum mechanical* EM fields to interact with the atoms, leading to "lase without inversion" [16]. When the system of three-level atoms, interacting with a classical and a quantum mechanical fields is placed inside a cavity with losses, it may be shown that the quantum noises associated with the system is reduced [17].

In the present work, we investigate the interaction of a three-level atom with two radiation fields for the following cases.

- i) *One* of the fields is *classical* while the other *one* is *quantum mechanical* in its coherent state.
- ii) *Both* fields are *quantum mechanical*, one of them being *regular* while the other one is *coherent*.
- iii) *Both* fields are *quantum mechanical* and *coherent*.

For each of the aforementioned cases, the atoms are initially assumed to be in their *ground state* or in a *maximal coherency*. For each mode of the EM fields and atomic initial conditions, the photonic mean numbers along with atomic level populations, as functions of time, are calculated. It is shown that for all cases the phenomenon of collapse and revival in the photonic mean numbers, as well as the atomic level populations occur. Another new result of this work is that when the atomic initial condition is that of coherency, the mean photon number is enhanced. For the case of initial atomic coherency, we show that the photonic mean number, as well as the atomic level populations, strongly depends upon the relative phase of the induced transition dipole moments. The physical reasons for such occurrence are also given.

The organization of this dissertation is as follows:

In chapter 2, the quantization of free electromagnetic field, in the Columb gauge is developed. We study a semiclassical and fully quantum mechanical theory of the interaction of a two-level atom with a single mode of radiation field in Chapter 3. The material of this chapter will be used extensively in the following chapters. In the fourth chapter, we introduce the atomic density operator for the three-level atoms and radiation fields. We then combine the two systems to find conditions under which the atomic populations are coherent. A study of the interaction between three-level atoms and two electromagnetic fields, one of which is classical, while the other is quantum-mechanical, is the subject of Chapter 5. We then compute the average number of photons and the atomic level populations. In the next chapter we investigate the