IN THE NAME OF GOD

APPLICATION OF COUPLED FINITE BOUNDARY ELEMENTS METHOD IN INVERSE HEAT CONDUCTION AND SOLIDIFICATION PROBLEMS

BY

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THESIS

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To my mother for her preying for me,

and to my wife for her patience and forbearance

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Abstract

Application of Coupled Finite Boundary Elements Method in Inverse Heat Conduction and Solidification Problems

By

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The solution to direct problems in engineering analysis has been studied by numerous researchers in various fields. In this respect different methods have been implemented. On the other hand, in comparison, the solution to inverse problems has been investigated only by a limited number of engineers or scientists.

In direct problems, geometrical domain, boundary conditions and domain loading are prescribed. The problem is solved to find one or several unknown variables within the domain or on the boundary. In inverse problems a boundary condition or a domain loading is unknown and by using some additional information within the domain or on the boundary, these unknowns are to be sought.

At first, it seems that by a rearrangement of unknowns and by a simple modification in direct formulations one can solve any inverse problem. The solutions obtained by such treatments are always severely oscillatory and

most of the time unacceptable. Therefore, some regularization methods must be employed to reduce the oscillations in the solution.

In this dissertation, various kinds of inverse problems in steady-state and transient heat conduction and solidification analysis are defined and solved. In the field of heat conduction, most of researchers have studied the inverse problems to find unknowns only on the boundary. However, in this work, new types of inverse problems which include the unknown intensity of the domain loading are studied.

The boundary element method for its clear advantages in such problems is used as the numerical method to formulate the problem. Coupled finite-boundary elements method is also used in some cases. Special mathematical treatments are implemented to evaluate the boundary integrals encountered in such problems. Also domain integrals are exactly transformed to the boundary to be evaluated with no need for domain discretization.

In the steady-state heat conduction, two categories of inverse problems have been defined. In the first category, the location and the orientation of heat sources are prescribed and intensity of sources are to be found by some additional information within the domain or on the boundary. In this respect a regularization algorithm based on the least squares method is used. In the second category of steady-state inverse problems the intensities of sources (point or line heat sources) are known and their locations and orientations must be found in a way that some desired conditions be satisfied. To solve this class of inverse problems a simple search method called "A Good Neighbor" has been devised.

In the transient heat conduction, similar to some cases in the steady-state heat conduction, inverse problems to find intensity of heat sources are studied. When only a few number of heat sources exist, a whole-domain regularization is employed and in cases where a large number of sources are considered and/or material properties are temperature-dependent and/or heat sources have movement, a sequential regularization is used. A secondary regularization algorithm is also presented and employed for better smoothing of solutions.

To solve inverse solidification problems a new method is presented in which the latent heat effects are implemented by introducing a pseudo heat source near the moving interface. In cases where material properties can be assumed constant, a boundary element formulation without domain discretization is used and in cases where material properties are temperature-dependent a coupled finite-boundary element formulation is employed. To reduce the oscillations in the solution of inverse solidification problems, sequential and secondary regularization algorithms are also used.

In each case several examples are presented to asses the accuracy of methods and algorithms. In respect to steady-state heat conduction the following inverse problems are presented:

- determination of heat intensity of three point sources
- determination of heat intensity of two line sources
- determination of location of one point heat source
- determination of location of two point heat sources
- determination of location and orientation of a line heat source

In the case of transient heat conduction analysis several inverse examples are also presented and solved which are entitled as:

- determination of time-dependent intensity of one heat source
- determination of intensity of heat sources in an electrical part
- determination of time-dependent intensities of three heat sources
- determination of time-dependent intensity of a moving heat source

Two other examples for inverse solidification analysis are also presented which are:

- an inverse solidification problem in a corner region
- an inverse solidification problem with temperature-dependent thermal conductivity

The obtained results show that the presented methods and algorithms are relatively accurate, stable and efficient.

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