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Josephson Effect in Ballistic Point Contacts Between Superconductors with f-Wave Triplet Pairing

Master's Thesis

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To:

My Dear Parents and
Nazila

Abstract

The stationary Josephson effect in point contacts between triplet superconductors is analyzed theoretically for the most probable models of the order parameter in UPt_3 and Sr_2RuO_4. The consequence of misorientation of crystals in superconducting banks is considered. It is shown that different models of the order parameter lead to quite different current-phase dependencies. For certain angles of misorientation the boundary between superconductors can generate the surface spontaneous current. In a number of cases the state with zero Josephson current and minimum free energy corresponds to a spontaneous phase difference. It depends on the misorientation angle and may take any value. We conclude that the experimental investigation of current-phase dependencies of small junctions can be used for determination of symmetry of the order parameter in mentioned superconductors.

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Chapter 1

Introduction

1.1 Introduction

Triplet superconductivity, which is the analogue of superfluidity in ^3He, was first discovered in heavy-fermion metal UPt_3 more than ten years ago [1, 2]. Recently the novel triplet superconductor Sr_2RuO_4 was discovered [3, 4]. If the fact of triplet pairing can be certainly determined, for example, in Knight shift experiments [5, 6], the identification of the symmetry of the order parameter is much more difficult task. A great deal of experimental and theoretical work done on UPt_3 and Sr_2RuO_4 are concerned with different thermodynamic and transport properties. But the precise order parameter symmetry still remains to be worked out (see, for example, [8, 9, 10, 11], and original references therein).

The calculation of the order parameter $\hat{\Delta}\left(\hat{\mathbf{k}}\right)$ in UPt_3 and Sr_2RuO_4 as a function of the direction of the momentum $\hat{\mathbf{k}}$ on a Fermi surface is a very complex problem. Nevertheless some general information on the order parameter can be obtained from the symmetry of the normal state. The superconducting state breaks one or more symmetries. The transition in the superconducting state implies the appearance of phase coherence which means the breaking of gauge symmetry. According to the Landau theory [7] of second order phase transitions, the order parameter is transformed only on irreducible representations of the symmetry group of the normal state. Conventional superconducting states have the total point symmetry of the crystal and belong to the unitary even representation A_{1g}. In unconventional superconductors this symmetry is broken. The parity of a superconductor with inversion symmetry can be specified using the Pauli principle. Because for triplet pairing the spin part of $\widehat{\Delta}$ is a symmetric second rank spinor, the orbital part must belong to the odd representation. The odd parity representations are described by spin-triplet order parameter of the form $\widehat{\mathbf{\Delta}}\left(\widehat{\mathbf{k}}\right) = i\mathbf{d}\left(\widehat{\mathbf{k}}\right)\cdot\widehat{\sigma}\widehat{\sigma}_2$, where the vectors $\widehat{\sigma} = \left(\widehat{\sigma}_1, \widehat{\sigma}_2, \widehat{\sigma}_3\right)$, and $\widehat{\sigma}_i$ are the Pauli matrices in the spin space. The vector $\mathbf{d}\left(\widehat{\mathbf{k}}\right) = -\mathbf{d}\left(-\widehat{\mathbf{k}}\right)$ is frequently termed the order parameter or gap vector of the triplet superconductor.

The symmetry point of view reserves considerable freedom for the order parameter in the selection of the irreducible representation and its basis functions. Therefore in many papers (see, for example, [8, 9, 10, 11, 12, 13, 14]) authors consider different models (the so-called scenarios) of superconductivity in UPt_3 and Sr_2RuO_4, which are based on possible representations of crystallographic point groups. The subsequent comparison of theoretical results with experiment makes it possible to conclude which symmetry the order parameter has.

In real crystalline superconductors there is no classification of Cooper pairing by angular momentum (s-wave, p-wave, d-wave, f-wave pairing, etc.). However these terms are often used for unconventional superconductors meaning that the point symmetry of the order parameter is the same as an appropriate representation of SO_3 symmetry group of isotropic conductor. In this terminology the conventional superconductors may be referred to as s-wave. For example, the "p-wave" pairing corresponds to the odd two-dimensional representation E_{1u} of the D_{6h} point group or E_u representation of the D_{4h} point group. The order parameter for these representations has the same symmetry as the superconducting state with angular momentum $l = 1$ of Cooper pairs in an isotropic conductor. If the symmetry of $\widehat{\Delta}$ cannot be formally related to any irreducible representation of the SO_3 group, these states are usually referred to as hybrid states.

Apparently, in crystalline triplet superconductors the order parameter has a more complex dependence on $\widehat{\mathbf{k}}$ in comparison with the well known p-wave order parameter for superfluid phases of ^3He. The heavy-fermion superconductor UPt_3 belongs to the hexagonal crystallographic point group (D_{6h}), and it is most likely that the pairing state belongs to the E_{2u} ("$f-$wave" state) representation. The layered perovskite material Sr_2RuO_4 belongs to the tetragonal crystallographic point group (D_{4h}). At first, the simplest "p-wave" model based on the E_u representation was proposed for the superconducting state in this metal. However this model was inconsistent with available experimental data, and in the papers [9, 10] few "f-wave" models of pairing state were suggested.

Theoretical study of specific heat, thermal conductivity, and ultrasound absorption for different models of triplet superconductivity shows a rather quantitative difference between the calculated dependencies[8, 9, 10, 14] . The Josephson effects are much more sensitive to the dependence of $\hat{\Delta}$ on the momentum direction on the Fermi surface. One of the possible types of superconducting weak links is the point contact between two massive superconductors. The microscopic theory of stationary Josephson effect in ballistic point contacts between conventional superconductors was developed in Ref.[15]. Later this theory was generalized for the pinhole model in ^3He [16, 17] and for point contacts between "d-wave" high-T_c superconductors [18, 19]. It was shown that current-phase dependence for the Josephson current in such systems is quite different from conventional superconductors, and states with a spontaneous phase difference are possible. The theoretical and experimental investigations of this effect in novel triplet superconductors seem to be interesting and able to distinguish among different candidates for the superconducting state.

In this thesis we theoretically investigate the stationary Josephson effect in the small junction between two bulk triplet superconductors with different orientations of the crystal axes with respect to the junction normal using quasiclassical approach. In Chapter 2 we bring an introductory discussion on quasiclassical theory. In Chapter 3 we describe the model of the junction and present a full set of equations and also the current density in the junction plane is analytically calculated for the non-self-consistent model of order parameter. In Chapter 4 the current-phase dependence for most likely models of "f-wave" superconductivity in UPt_3 and Sr_2RuO_4 is analyzed for different mutual orientations of the banks. We end in Chapter 4 with the results and some conclusions.

1.2 Josephson effect

The effects related to a weak coupling between two phase-coherent quantum systems are often collectively termed Josephson phenomena. The name dates back to Josephson, who first predicted such effects (tunneling of Cooper pairs) to take place in a weak connection between two superconductors, a so-called Josephson junction [20]. The name "Josephson junction" is usually reserved only for the pure tunneling junctions in superconductors, and microbridges or superconductor junctions etc. are put under the more general category of "weak links" [21].

1.2.1 Basic concepts of superconducting weak links

Imagine a container of superconductor, divided into two parts (1 and 2) by a thin membrane. The two condensates are described by some macroscopic wavefunctions, or order parameters, which have well-defined but different phases ϕ_1 and ϕ_2. We assume that there are no other relevant degrees of freedom. The two sides are then weakly coupled by introducing some (sufficiently) small orifice(s) in the dividing membrane. If we denote the phase difference between the condensate wavefunctions by $\phi = \phi_1 - \phi_2$, then there will be a supercurrent flowing through the weak link, given by the simple formula

$$I(\phi) = I_c \sin\phi, \tag{1.1}$$

where I_c is a critical current specific to the junction. If we apply a finite voltage difference ΔV between the two sides keeping the temperature constant, there will also be a difference in the chemical potentials $\Delta\mu = e\Delta V$, e being the charge of electron. The well-known Josephson-Anderson phase-evolution equation [23]

$$\frac{\partial\phi}{\partial t} = -\frac{\Delta\mu}{\hbar} \tag{1.2}$$

then tells that ϕ will change in time. As a result of this and the periodic form of Eq. (1.1), a constant $\Delta\mu$ results in an oscillating supercurrent through the weak link.

The current-phase relation (1.1) is accurate only for pure tunneling junctions. For micro-bridge-type weak links there are deviations from the sine form [21, 22]. Generally at lower temperatures, or stronger coupling, the sinosoidal will become slanted. The length scale determining the effective size and thus the strength of the coupling is always the superconducting coherence length, which grows with temperature and diverges at the transition. The problem of a pinhole between two usual (s-wave) superconductors was first discussed by Kulik and Omelyanchouk [15], a good review of which can be found in Ref. [24].

1.3 Symmetry of the order parameter

In this chapter we bring a review on basic characteristics of heavy fermions and how their order parameter as unconventional superconductors can be constructed from different experiments [26].

The study of heavy fermion (HF) superconductivity began more than 20 years ago with the discovery of superconductivity in $CeCu_2Si_2$[27]. At that time the major mystery was how superconductivity could be supported in a system with strong local moments on the Ce ions. Since that time the field of HF superconductivity has expanded to include five (possibly six) new uranium-based superconductors at ambient pressure and one new rare-earth-based superconductor at elevated pressures. Collectively, these materials display a rich variety of unexpected properties, which have enhanced our knowledge, not only of superconductivity, but also of the general behavior of highly correlated electrons in metals. It is interesting to note that the original question posed above for the discovery of $CeCu_2Si_2$, namely the coexistence of magnetism and superconductivity in HF materials, remains a central challenge for the understanding of these materials today. This section is devoted to discussing important properties of heavy fermions using which symmetry of their order parameter can be constructed and determined among various candidates. The scope and depth of this section is intended to provide an easy, but stimulating experience for readers familiar with the basics of superconductivity. More comprehensive reviews of HFs has been given by Grewe and Steglich, Ott and Fisk, and Hess, Riseborough and Smith [27]. Normal state properties were reviewed in an earlier article in this series by Lawrence and Mills [28].

1.3.1 Normal-state properties

The starting point for understanding superconductivity in a material must be a study of the normal ground state from which the superconducting state emerges. We have brought those normal-state properties in Table(1.1) which are most useful in establishing a picture of the superconducting state. The high-temperature ground state in HFs is reasonably well understood on the basis of extensions of single-ion Kondo theory [28]. How the variety of low temperature ground states (paramagnetic, magnetic, semiconducting and superconducting) evolve from the high-temperature normal state is only partially understood, however, although the following considerations are generally agreed upon. There is a crossover from local-moment behavior at high temperatures to a reduced-moment regime at low temperatures, where the f-electron moments are reduced to a fraction of their high temperature values. This compensation of the high-temperature f-moments occurs through an antiferromagnetic exchange interaction which produces a virtual bound state between the f-moments and the conduction electrons. In dilute metals this is the well-known single-ion Kondo interaction. Although this crossover from full- to reduced-moment behavior does not involve a phase transition, it can still be characterized by a coherence temperature $T_{coh} \approx 1 - 100K$.

The strong exchange coupling between the conduction electrons and the f-electrons also leads to a large resistivity in HF materials, typically several hundred $\mu\Omega$-cm at room temperature. The resistivity increases as the temperature is decreased below room temperature, reaching a maximum at T_{max} and then dropping drastically below $T_{\max}$, indicating the loss of inelastic scattering and the onset of the HF state. The normal-state resistivity at temperatures just above the superconducting transition temperature T_c is typically only a few $\mu\Omega$-cm. At $T = T_{coh} << T_{max}$, if no magnetic ordering occurs, the system enters a Fermi liquid-like state with strongly renormalized quasiparticles of large effective mass $m^* \approx 100m_e$. This state is characterized by a large linear coefficient of specific heat, indicating a high density of states at the Fermi surface, and a resistivity which varies as T^2 .

Whether HFs exhibit an antiferromagnetic or paramagnetic ground state depends upon a competition between the local on-site exchange interaction, which