Allameh Tabatabai University
E.C.O. College of Insurance

# “A benchmarking approach to optimal asset allocation for insurers and pension funds”

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
MASTER OF SCIENCE
IN THE DEPARTMENT OF ACTUARIAL SCIENCE

**Supervisor:** Dr. A.T.Payandeh
**Advisor:** Dr. Gh. Mahdavi

**MAHBUBE GOLESTANI**

# ACKNOWLEDGEMENTS

I would like to express my deep and sincere gratitude to my thesis supervisor, *Dr. Amir Timor Payandeh* for his understanding, guidance, motivation and encouragement throughout this study. Also I would like to thank my advisor, *Dr.Ghadir Mahdavi* for his constructive comments and useful notes about my research.

I also wish to express my loving thanks to my parents and brother who always believe in me more than I do, without their encouragement and understanding it would be impossible for me to finish this work.

Lastly, I offer my regards and blessings to all of those who supported me in any respect during the completion of this project.

# Abstract

Uncertainty in the financial market will be driven by underlying Brownian motions, while the assets are assumed to be general stochastic processes adapted to the filtration of the Brownian motions.

The goal of this study is to calculate the accumulated wealth in order to optimize the expected terminal value using a suitable utility function.

This thesis introduced the Lim-Wong's benchmark functions as utility function for such optimization. We employ Lim-Wong's benchmark function to solve the asset allocation problem. Some advantages of the Lim-Wong's benchmark approach have been given.

Our approach tolerates but progressively penalizes underperformance, and progressively rewards outperformance. A general solution under general market models, benchmarks, and concave benchmarking functions is presented, and insights to the impact of benchmarking to the optimal portfolio are obtained.

Application of the Lim-Wong's benchmark function to an investment in Iranian Stock market and Iranian bank system is given. Using a panel data (1381-1391) collected from Stock Alborz Insurance and estimating asset allocation as option, then find optimal wealth strategy derived from asset allocation. Finally, we compared optimal wealth as Lim Wong's benchmark with terminal wealth portfolio as non-benchmark.

Considering if stock price is less then strike price, terminal wealth as benchmark equal non benchmark. As to stock price is more than strike price terminal at first terminal wealth of the state non-benchmark is higher, then the terminal wealth as benchmark is higher.

**Keywords**: asset liability management, portfolio optimization, Lim-Wong`s benchmarking.

**JEL classification**: C61, G11.

# Contents

## 1. General Introduction



## 2. Mathematical Foundations

# LIST OF TABLES



# LIST OF FIGURES

# Chapter 1

## 1. General Introduction

### 1.1 Introduction

Pension funds are one of the most important institutions in financial markets due to their large capacity of investment. They also fulfill an important function in that pension companies complement the role of the government, allowing those workers who have reached retirement age to maintain a reasonable standard of living. From a general view point, regarding to the risk assignment, there are two principal alternatives in pension plan designation: (1) Defined Contribution & (2) Defined Benefit.

In the defined benefit plan, the pension fund guarantees to pay the employee in retirement a fixed monthly income for life.

While in the defined contributions plans are those in which the employer agrees to contribute a fixed amount to the employee's pension fund each year.

The income that the employee receives during retirement depends upon how much money the plan had accumulated and how much income that amount can sustain. From a historical perspective, defined benefit plans proved to be more popular than defined contribution plans. Nowadays, most of the plans created in pension funds have been based on defined contributions, such as Appropriate Personal Pension in UK and Individual Retirement Accounts in USA (Boulier, 2001).

There is another pension fund scheme, called the pay as-you-go, say PAYG system. The PAYG is a scheme where workers pay contributions to the fund while pensioners receive their pensions, which are out-flows from the fund wealth.

In France, the pay-as-you-go system was efficient in the past, but actually it is limited by the demographic and the economic situation. Indeed, the age structure of the population and the ratio of the working-age population show the limits of this system. The ratio of retirees to workers is about 50% in 2010. In 2040, this number will arrive at 70% if the retired age does not change (Fitoussi, 1999).

However, no enabling legislation has been enacted. Nevertheless, the foundations for private pension funds have been laid, and the financial community must be ready to bring retirement products to the market at the appropriate time. Traditional optimal asset allocation problems in the investment management typically imply maximization of the expected utility of a terminal portfolio value, and where the utility is a concave function that satisfies some properties.

However it is well known (Yang & Zhang, 2005) that it is also important to consider the effects of the liability payments when determining the optimal asset allocation for insurers and pension funds. Moreover, adaptation of standard financial economic methods (for example, maximization of expected power utility of terminal surplus) in an insurance/pensions setting is often not straightforward due to the nature of insurance and pension problems. In particular, typical utility functions favored in the financial economics literature (e.g. power utility) often involve a strict floor (typically at zero surplus) in the terminal wealth which cannot be violated for a related approach where the solvency guarantee is placed in the constraints (Tepla, 2001). While such absolute solvency guarantees may be a desirable property in some applications, it is well recognized in the insurance literature that such guarantees may not be financially desirable, or, feasible. For example, in underfunded defined benefit pension plans the assets are not sufficient to cover the liabilities with certainty by definition (the discussion in Detemple & Rindisbacher, 2008), and in many insurance models the probabilistic structures of many claim processes often mean that liabilities cannot be funded with certainty under a realistic initial asset wealth, even if a guarantee is achievable at all. Such situations lead to an ill-posed optimization problem if the floor cannot be met with certainty, and no optimal solutions are available.

We can also turn around and look at the effects or impacts of variables such as liabilities, inflation, recession, stagflation and even guarantee on pension investment. Some of these factors are generally known as uncertainties. Then, the idea of benchmarking is very convenient in this case. The use of benchmarking has become a common approach for enhancing the performance of companies. Thus, when applying benchmarks, firms compare their own activities and performance to those of other, appropriately selected, comparable organizations. The purpose of benchmarking varies from one company to another and it helps in determining the true source of performance.

Several methods have been proposed in the insurance literature to consider the optimal asset allocation problem with tolerance for possible shortfalls. We note, however, that these methods differ from ours in either the objective function that is used and/or the stochastic model for the assets and liabilities. The objective functions that have been studied in the literature can be broadly classified into two alternative groups. The first group considers as an objective the probability of reaching some desired outcome (for example, solvency). Examples include Browne (1995, 1999, 2000), Josa-Fombellida & Rincón-Zapatero (2006). From a behavioral perspective however this is often undesirable as it implies that the decision maker will solely focus on achieving the desired outcome (for example, to stay solvent), and in particular the level of final outperformance or underperformance is ignored. The second approach is to maximize the expected utility of the surplus.

In order to tolerate shortfalls such an approach requires a utility function that can be defined over negative values. The two prime examples of such utility functions are those of exponential and quadratic types. The application of an exponential utility function in dynamic asset-liability modelling was explored recently in Yang and Zhang (2005), Wang (2007) and Wang et al. (2007). Quadratic utility/loss functions have been explored in the insurance literature by Cairns (2000), Chiu & Li (2006), Xie et al. (2008), Chen et al. (2008), and is also related to the problem of mean-variance hedging. While the use of the exponential and quadratic approaches provide a number of benefits including tractability, both have undesirable properties which are well known. Exponential utility, for example, implies constant absolute risk aversion, while quadratic utility possesses a saturation level beyond which utility decreases with increasing wealth.

The use of a benchmarked return in a ``utility" function has also been used in the literature for problems related to real returns and also with target pension funding ratios. In particular, Brennan & Xia (2002) and De Jong (2008) considered a dynamic asset allocation problem for long term investors where the objective function is a power transformation of the real wealth, while Cairns et al. (2006) investigate stochastic life styling strategies for pension plans by considering as an objective function a power transformation of the terminal wealth divided by terminal salary.

Davis & Lleo (2008) considered a risk sensitive asset allocation problem where the objective is related to the power ``utility" of the benchmarked return, and where the benchmark is a variant of Geometric Brownian Motion. We note however that the previous papers assume that prices processes and benchmarks are variants of

Geometric Brownian motion, and power "utility" functions. In contrast, we consider very general dynamics for asset price dynamics and the benchmark, and general increasing concave functions of the relative performance. In particular, benchmarks such as the maximum of a random quantity (such as a stock index) and a minimum return, which are not naturally formulated as Geometric Brownian Motion but are of interest in asset and liability management, can be handled in our model.

Some of the major problems associated with benchmarking are dealt with in this thesis including the risks involved and measures taken to deal with the inherent risk, such as risk adjustment. In fund management, benchmarks are used as a guide to improvement, and fund managers are assessed in terms of their relative performance against the benchmark. Benchmarks are based on an objective consideration of the needs of fund managers. They can change with the environment.

The plan member in a classical defined contribution-pension plan experiences a risk linked to inflation which could amount to substantial losses. The pension manager must ensure that the benefit from non-inflation-linked pension will be sufficient to cover the future expenses as prices will have increased due to inflation.

A given annual inflation rate of 3% over 35 years will certainly reduce the real value of $200,000 to $ 71,077 today. The second problem that the fund manager faces here is that the number of workers may decline due to recession. This is known also as the demographic risk. It is pertinent that the pension manager must link the fund to inflation in order to reduce his risk. Inflation linked products include for instance inflations puts, calls, swaps, floors and caps (Beletshi, 2006).

This thesis considers a model for managing pension funds with benchmarking in an inflationary market and we extend our framework to cover the following:

1. Interest rates and inflation, and we follow a continuous time approach.

2. The financial market consisting of three assets:
   - a money market account
   - a classical stock
   - an inflation-linked bond.

3. The stochastic behavior of inflation in the financial market.

We assume that the money market account and the stock are specified as in the classical Black-Scholes model, while the Stock price will be specified as a geometric Brownian motion. The time horizon of the pension fund management is denoted by [0, T].

## 1.2 Overview of research problem and background

The world economy is expected to grow bigger in different aspects everyday and investors have to be more concerned about movements in the economy. The inflation market is also expected to develop exponentially, and the number of participants is of course also growing. Cyclical economic events such as inflation, stagflation and recession can affect investments. Economists define inflation as a general and a progressive rise in prices. The recession is generally characterized by the unemployment when the state of the economy declines. Stagflation is an inflationary employment situation or an inflation-recession event. The challenge for the future is to include more strategies, techniques and skills in our management that would allow participants to always be able to hedge against this cyclicality. Several studies should be considered models with jumps and other sources of incompleteness, models with inflation and recession. Another emerging challenge would be to create a model set of life-cycle pension funds, which can serve as benchmarks against which the performance of pension fund managers can be measured. Several works have been undertaken on inflation, recession, interest rates and portfolio management. These notions were studied extensively in economics and especially in macroeconomics. Nowadays these notions merge competences, skills and knowledge in the field of statistics, mathematics and finance. In macroeconomics, Fisher was a pioneer on the theory of interest rate (Fisher, 1930). He formed his famous hypothesis that the nominal interest rates should vary closely with the movements in expected inflation. This hypothesis connects two distinct parts of economy which are: inflation which expresses changes in the supply-demand conditions on the commodity spot market while nominal interest rates reflect differences in supply-demand conditions on the money market. Fisher's hypothesis was supported by several empirical studies either by using available survey data or by analyzing market data on inflation-linked bonds in the UK, the US or Canada. In their empirical studies Ang & Bekaert (2008) investigate the connection between nominal and real interest rates and inflation. Changes in nominal interest rates must be due to either movements in real interest rates, expected inflation, or the inflation risk premium.

Empirical research does not support the hypothesis that rates of inflation are constant over time. The variability of yearly measured inflation rates varied widely over the last century. There was some evidence that high variability can be associated with periods of high inflation. From an economic point of view, analyses of interest rate volatility can also be used to gauge inflation rate variability, since the level of interest rates provides an indicator of inflation expectations. Besides Fisher's hypothesis there exist a variety of different models describing the relation between nominal interest, real interest and inflation such as Taylor's rule and the forward rate rule (Gerlach & Schnabel, 1999).

## 1.3 Research Objectives

The main objective of the pension company is to increase the expected utility function of the relative performance of the asset portfolio compared to a given benchmark by investing strategically in the stock and the money market account.
The benchmark can be either any target to attain or any ratio to be compared with the initial wealth or the final wealth of the pension company or against the pension company's asset allocation. The pension company would be required to compare his allocation or his wealth to the level of the benchmark at different time ticks during of the management process. The benchmark entity can also be a completely independent basket of commodities which is published monthly and which the pension company has adopted, in agreement with pension members. The main strategy to hedge against the risk associated with his investment or management, is by selling inflation-linked bonds and by also revising his investment strategy with the benchmarks. Thus, we can construct appropriate stochastic models which are subject to follow a restriction from a geometric Brownian motion. We then extend our study to interest rates and inflation in a continuous time approach, and to the pricing formula for inflation-linked bonds with the support of a European call option on inflation.

## 1.4 Research Problem

Asset allocation is an investment portfolio technique that aims to balance risk and create diversification by dividing assets among major categories such as cash, bonds, stocks, real estate and derivatives.

A benchmarking approach to optimal asset allocation for insurers and pension funds has not been studied before in Iran. Related works can be found though and there is a series of publications which focus on modeling the inflation process using the Fisher equation. Important contributions in continuous time are due to Blake et al. (2001) and Cairns (2006). In Blake (2001) and Deelstra (2002) used stochastic dynamic programming to solve the optimization problem.

The goal of the fund manager in these studies is to invest the accumulated wealth in order to optimize the expected terminal value using a suitable utility function.

This thesis, the optimal asset allocation of pension funds using a benchmarking approach has been studied by Lim & Wong (2010). In their study, the objective functional is an increasing function of the relative performance of the insurance company's asset portfolio compared to a benchmark. Contrary to their study, our study consists of a model with benchmark in an inflationary market. We use an approach similar to this under which the objective of the pension company is an increasing function of the relative performance of its asset portfolio compared to a benchmark. The first step, selecting a particular symbol which its stock traded in Iranian stock market. It should be noted that collection of asset base (stock price) should be applicable in conditions Black-Scholes. After calculating portfolio allocation strategy of the Lim & Wong`s benchmark, we find optimal wealth strategy derived from asset allocation. Finally, comparing optimal wealth as Lim & Wong`s benchmark with terminal wealth portfolio as non-benchmark.

## 1.5 Research purpose

We examined the effects of Lim-Wong's benchmark on the option using Iranian Stock Market.

## 1.6 Research hypothesis

This research aims to examine the following hypothesis in the Iranian financial market:

**Hypothesis 1 )** Lim–Wong’s benchmark is a convenient criterion (from calculation viewpoints and simple assumptions to check) to calculate company terminal wealth.

**Hypothesis 2 )** Lim-Wong's benchmark is an appropriate in regard of non-benchmark, whenever the stock price is higher than the strike price.

**Hypothesis 3 )** Lim-Wong's benchmark works as well as non benchmark, whenever the stock price is less or equal to the strike price.

## 1.7 Research assumptions

**assumption 1 )** We want to invest the capital pension fund in 2 financial markets which one has a constant interest rate and the second one has stochastic interest rate.

**assumption 2 )** Stock prices should be treated under a geometric Brownian motion process and following as log normal distribution.

## 1.8 Structure of the thesis

This thesis has been structured into five chapters including on introduction.
Chapter 2 deals with mathematical preliminaries on stochastic calculus used throughout our thesis. We present some stochastic concepts, such as Brownian motion, martingales, utility function, stock and money market and other. Chapter Three introduces the financial market and defines a benchmark, and provides discussion regarding the concept of a benchmarking function and its relationship to standard utility functions. A general solution to the optimization problem using martingale techniques is presented. In Chapter 4 provides a numerical application and shows the qualitative behavior of the benchmarking asset allocation strategy. We summarize the main results in Chapter 5.

# Chapter 2

## 2. **Mathematical Foundations**

### 2.1 Introduction

In this present chapter, we record some useful mathematical background material, used throughout in this thesis. We define concepts such as random variables, stochastic processes, Martingales, Brownian motion, etc., and give some basic results. Our main references on such basics are Etheridge (2002), Hull (2007 ) and Karatzas et al (1987).

### 2.2 Pension Found

What is a pension? A *pension* is a stream of payments that starts when someone retires and continues in payment until they die[1]. In other words, a pension provides lifetime income security in retirement for however long the retiree lives (Bodie, 1990). A pension therefore has two essential purposes. The first is *consumption smoothing* over an individual's lifecycle, i.e., a pension provides an income in retirement when someone is no longer working in exchange for contributions into a

[1] . This is also the definition of a life annuity, so a pension is an example of a life annuity.

pension scheme when they are. The second is *insurance*, especially in respect of longevity risk – the uncertainty attached to an individual's length of life.

Public policy might have two additional objectives for a pension scheme. The first is *poverty relief*: a society might wish its pensioners to have a minimum standard of living in retirement. The second is a *distributional objective*: a society might wish to distribute additional resources above the poverty level to certain members of society, such as women bringing up children and other careers (Barr, 2004).

There are only two ways of 'paying' for a pension. In the first case, young workers agree to pay (out of their labor income) the pension of retired people in return for the promise that the next generation of workers pays for their pension. This is called an unfunded or pay-as-you-go (PAYG) pension scheme or plan. In the second case, each generation of workers saves (out of their labor income) for their pensions in a funded pension scheme or plan. There are two key features of both types of pension arrangements: time and risk. Workers must pay now for something that they will get in the future, i.e., the pension provides the economic function of transferring income (and hence consumption) from work years to retirement years. There is some risk that the actual pension payments received will be less than those expected when the plan was first started. Indeed, there is some chance that the pension might not actually be paid at all on account of the pension scheme becoming insolvent. This is why people talk about a pension promise, rather than a pension guarantees (Blake, 2006).