

IN THE NAME OF GOD

***STUDY OF THE DEEP INELASTIC  
ELECTRON-PROTON SCATTERING  
AND THE PARTON MODEL OF  
PROTON***

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*Dedicated to:*

*My dear mother*

W/P RY

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# ***ABSTRACT***

## ***STUDY OF THE DEEP INELASTIC ELECTRON-PROTON SCATTERING AND THE PARTON MODEL OF PROTON***

By  
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In this research, starting with an introduction to Dirac equation and quantum electrodynamics, the inclusive cross section formula for the deep inelastic electron-proton scattering at high energies are derived. Then by comparing these theoretical calculations with the results obtained from the scattering experiments, the two structure functions  $W_1$  and  $W_2$  which contain all the information about the inner structure of the target protons are extracted. Then the scaling behavior of the structure functions is interpreted as an experimental evidence for the existence of point scatterers inside the proton which are actually good candidates for "quarks".

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# **CHAPTER 1**

## ***Introduction***

From the very old times human beings have had a tendency to simplify the complicated world around them. People would like to imagine the different and various existing materials and objects of the universe as different compositions of a smaller number of more fundamental entities or elements. They actually tend to have a short and lucid answer to the question, what the universe is made of. The first attempt of this kind was the Anaximenes' theory of four elements: Air, Fire, water and Earth. His theory says that everything we see in everyday life is a combination of these four fundamental elements.

Another noticeable and more successful simplification was made by Mendeleev some 25 centuries later. He sort of extended the previous picture and introduced the periodic table which contains over 100 elements. Eventhough Anaximenes model is conceptually superior for its simplicity and economy in number of building blocks, scientists prefer that of Mendeleev because the former has a serious problem. It does not make sense when considered not only with scientific experiments but even also with everyday life experiences. Therefore they would say it is wrong. Mendeleev's answer is right but the number of its elements is too large in order for the model to be desirable as an ultimate and fundamental solution.

The apparently regular organisation of the table together with the quantized mass spectrum of the elements pointed the way to perceive particles called *protons* and *neutrons* (collectively known as *nucleons*) as the substructure for atomic nuclei which are formed when nucleons are glued together with the strong or nuclear force. Nuclei subsequently bind with the electrons to produce the atoms of chemical elements.

This was a comfortable situation. We had a theory looking at the world as combination of an astoundingly small number of elements. Just three; electrons, protons, and neutrons. But this comfort did not last long. With the development of particle accelerators it was soon observed that there exists a set of many other strongly interacting fermions of which proton and neutron are only the lightest members. Another sequence of bosons were also discovered. *Fermion* is a term for particles having half-integral spins and *Boson* for those with integral spins. Since fermions have larger masses they are also called *baryons*. Bosons which are relatively lighter are also called *mesons*. Baryons and mesons are known collectively as *hadrons*.

In almost the same way as the arguments for composite atoms based on Mendeleev's table, the proliferation of these particles showed the way to the substructure of hadrons (the *quarks*). In this new theory, introduced by M. Gell-mann, all baryons are composed of three quarks and each meson is a composite of a pair of quark-antiquark [1].

Quarks are of two kinds, *up* quark (*u*) with charge  $+2/3$  (in unit of positron charge *e*), and *down* quark (*d*) with charge  $-1/3$ . Each appearing in three colors; Blue, Green, and Red. Each

quark ( $q$ ) has an antiparticle or *antiquark* ( $\bar{q}$ ) with opposite charge and complement color. All quarks and antiquarks are fermions (have spin 1/2).

Furthermore note that these quarks and antiquarks introduced so far are the first generation of quarks. there are two other collections with similar members and characteristics but different only in mass range known as second and third generations.

Peculiarities stated above for quarks, combined with a manipulated rule that constrains the free particles to be white or colorless, can provide possibilities to construct not only all of particles discovered by the time but also to predict the existence of some other yet uncovered particles, as it really did for  $\Lambda^-$  for example and it was actually discovered afterwards. According to quark model, the quark contents of proton and neutron for instance are "uud" and "udd" respectively.

To complete the picture I should say that our today's answer to the original question is that, in addition to the 36 quarks (and antiquarks) there are three generation of *leptons*. The first one contains the electron ( $e^-$ ) and the electron-neutrino ( $\nu_e$ ) and their antiparticles, positron ( $e^+$ ) and electron-antineutrino ( $\bar{\nu}_e$ ). These leptons add up to 12. The rest of elementary particles are *mediators* which are interchanged between quarks and leptons while interacting with each other. The *photon* for the electromagnetic force, " $W^\pm$ " and " $Z$ " particles for the weak interaction, *graviton* (presumably) for gravity, and eight *gluons* for the strong interaction[2].

Despite the great compatibility of this ultimate model, physicists wanted to make sure about the existence of quarks by designing some experiments to show them as directly as possible. This was why the *deep inelastic* electron-proton scattering experiments were carried out around 1960. The present research is a review of theoretical tools and gained experimental evidences for the existence of quarks.

For this purpose some useful aspects of scattering cross section and Fermi's golden rule is introduced in the next chapter. In chapter 3, starting with the Dirac equation and its solutions, Feynman rules for quantum electrodynamics and calculation of amplitudes of interactions are considered. This part will be the basis of the theory used in this research. Using the materials and methods developed in chapters 2 and 3, elastic and inelastic scattering cross sections are derived in the fourth chapter. Finally in the two last chapters the deep inelastic  $e$ - $p$  scattering experiments and their results are discussed in some detail.

## ***CHAPTER 2***

### ***Cross Sections and Golden Rules***

Having a look at our future needs, we start with an introduction to the physical meaning of cross sections and then to Fermi's golden rule and its relation to the scattering cross sections

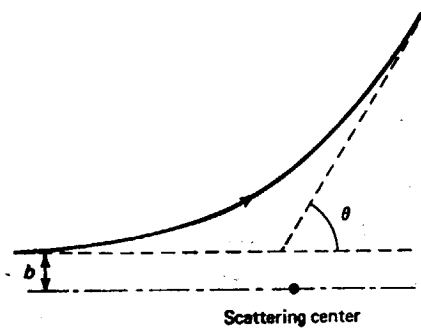
#### ***2.1 Cross Sections***

What is the scattering cross section? What is its physical meaning and interpretation, and for that, what quantity should the experimentalists measure and the theorists calculate? In the case of an archer aiming at plates hung on a wall the parameter of interest would be the size of the target, or more precisely the cross sectional area it presents to the stream of incoming arrows. For this case the experimentalist measures the number of arrows that hit a plate divided by the total number of arrows if they are sufficiently numerous, or equivalently the probability of a single arrow to hit a plate. And the theorist would calculate the sum of areas of the plates divided by the total area headed for by the stream of arrows.

In a crude sense, the same goes for elementary particle scattering. If you fire a stream of electrons into a tank of hydrogen (which is essentially a collection of protons), the parameter of interest is the size of the proton, the cross sectional area it presents to the incident beam. However the situation is more complicated than in archery for several reasons. First of all the target is "soft". It is not a simple case of "hit-or-miss", but rather the closer you come the greater the deflection and so on.

Let's consider the question of what we mean by a cross section when the target is soft. Suppose a particle (maybe an electron), comes along, encounters some kind of potential (perhaps the coulomb potential of an stationary proton) and scatters off at an angle  $\theta$ . This *scattering angle* is a function of the *impact parameter*  $b$ , the distance by which the incident particle would have missed the target center, had it continued on its original trajectory (fig 2.1). Ordinarily, the smaller the impact parameter, the larger the deflection, but the actual form of  $\theta(b)$  depends on particular potential involved, but there is a general formula for that.

If the particle comes in with an impact parameter between  $b$  and  $b+db$ , it will emerge with a scattering angle between  $\theta$  and  $\theta+d\theta$ . More generally, if it passes through an infinitesimal area  $d\sigma$ , it will scatter into a corresponding solid angle  $d\Omega$ , (Fig. 2.2). Naturally, if we make  $d\sigma$  larger,  $d\Omega$  will



**Figure 2.1** Scattering from a fixed potential:  $\theta$  is the scattering angle,  $b$  is the impact parameter.

become proportionally larger. The proportionality factor is called the *differential scattering cross section*,  $D$ , or equivalently  $d\sigma/d\Omega$ :

$$d\sigma = D(\theta) d\Omega \quad (2.1)$$

In principle,  $D$  might depend on the azimuthal angle  $\phi$ ; however, most potentials of interest are spherically symmetrical in which case the differential cross section depends only on  $\theta$  (or preferably on  $b$ ).

Now from figure 2.2 we write:

$$d\sigma = |b db d\phi| \quad , \quad d\Omega = |\sin\theta d\theta d\phi|$$

Hence,

$$D(\theta) = \frac{d\sigma}{d\Omega} = \left| \frac{b}{\sin\theta} \left( \frac{db}{d\theta} \right) \right| \quad (2.2)$$

### **Example 2.1** Rutherford Scattering

A particle of charge  $q_1$  scatters off a stationary particle of