بسم الله الرحمن الرحيم

IN THE NAME OF GOD

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G-Logics and Multiple-Valued Logics Based on Hazy Structures

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Abstract

In this thesis we present a study of the G-set signed formulas which is an extension of set signed formulas and introduce the G-reduction rules to provide G-logics. We will extend the corresponding Hintinkka and Kalmar downward and upward saturated collections of G-set signed formulas to establish the completeness of G-logics. Instead of considering G-sets or convex intervals, we introduce and impose the convex hazy structures on the lattice of truth values to investigate the indistinguishable formulas in a multiple-valued logic. The Haz-necessities and Haz-possibilities are also introduced to study some of their properties.

Finally, we introduce three new degrees of similarity between two fuzzy sets. We will then define the validity measures, by applying these degrees to expert's views about the linguistic variables corresponding to the content validity of the questions in any questionnaire.

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0.1 GENERAL INTRODUCTION TO G-LOGICS AND MULTIPLE-VALUED LOGICS BASED ON HAZY STRUCTURES

Any convex interval in a (bounded) lattice, with zero and unit elements 0,1, respectively, is a G-set. Generally, in a multiple-valued logic with a matrix $\underline{\underline{\mathcal{M}}}$, a finite domain K, and designated set D, the statement that, a formula φ is not satisfiable is, in fact, equivalent to the statement that there is no valuation v, such that $v(\varphi) \in D$. This, again is equivalent to the statement that for any valuation v, $v(\varphi) \in K - D$. So the set $(K - D)$ may be considered as the sign for such a formula φ. Since D is always of the form of a principal filter in K, so we get the motivation for using G-set as the signs of formulas. G-logics are based on the G-sets as the signs of formulas in multiple valued propositional or first order logics.

0.2 SUMMARY OF RESULTS

The main results are as follows:

1) A lattice structure is imposed on any GK (defined later) and its well-foundedness is proved.

2) G-reduction rules, G-logics, and the notion of provability are introduced and in light of some examples, the set-reduction rules of Hähnle for 3-valued Lukasiewicz, are to be obtained from those of corresponding G-reduction rules.

3) For any homomorphism $f \in Hom(K_1, K_2)$, it is proved that the set of equivalence classes of GK modulo f is isomorphic to GK_2.

4) The soundness and completeness for G-logics are proved.

5) Hazy structures are imposed on the lattice L, of truth values, instead of convex intervals, and by a very nice theorem it is proved that the new structure preserves the inherited lattice structure of L, into the new neighbourhood space as the truth values.

6) The relation of τ-indistinguishability is defined and it is proved that this relation is reflexive and symmetric, but not transitive.

7) As a very interesting result, we prove the existance of an m-valued logic, which is in-fact a reduced form of n-valued logic, for arbitrary ordinals m and n, with $2 \leq m \leq n$.

As a simple, but very important result of this theorem, is that of the reduction of continuum-valued logic, to any m-valued logics ($m \leq \omega_1$), in particular to a Lukasiewicz 3-valued logics.

8) Several properties of τ-necessities and τ-possibilities are investigated and proved. Also some of the known and new properties of the notion of duality, are proved for the dual of K-fuzzy sets.

9) Several methods of computing the validity measures are discussed in which we are using the experts' views about the linguistic variables.

0.3 FUTURE RESEARCH DIRECTIONS:

We do only mention here, the possibility of the extesion of G-logics to hazy-logics, via introducing hazy neighbourhoods as the set signs for formulas, and selecting the appropsiate hazy-reduction rules parallel to the corresponding G-reduction, and investigating the soundness and completeness of the new hazy-logics.

0.4 Notations And Conventions:

Basic set-theoretic and lattice operations are used without any comments. The list of symbols are as follows:

ϕ	the empty set
S, s	$S \cup \{s\}$
$Sub(X)$	the power set of X
$\mathcal{Z}$	the set of integers
$\underline{m}$	the set $\{0, 1, \cdots, m-1\}$
$\mathcal{F} = \{f_i\}_{i \in \omega}$	an indexed family of operations
$\underline{\underline{\mathcal{A}}} = \langle A, \mathcal{F} \rangle$	an abstract algebra
$\mu \in \omega^{\omega}$	the type of $\underline{\underline{\mathcal{A}}}$
$\{\mu(1), \cdots, \mu(r)\}$	the signature of $\underline{\underline{\mathcal{A}}}$
$h : \underline{\underline{\mathcal{A}}} \overset{Hom}{\rightarrow} \underline{\underline{\mathcal{B}}}$	a homomorphism h from $\underline{\underline{\mathcal{A}}}$ into $\underline{\underline{\mathcal{B}}}$
$h \in Hom(\underline{\underline{\mathcal{A}}}, \underline{\underline{\mathcal{B}}})$	a homomorphism h from $\underline{\underline{\mathcal{A}}}$ into $\underline{\underline{\mathcal{B}}}$
$L_0 = \{p_i \mid i \in \omega\}$	a denumerable set of propositional (atomic) formulas
L	set of propositional formulas

$\underline{\underline{L}} = \langle L, \mathcal{F} \rangle$	propositional language
$\underline{\underline{\mathcal{K}}} = \langle K, \mathcal{F} \rangle$	an algebra similar to $\underline{\underline{L}}$
$\underline{\underline{\mathcal{M}}} = \langle \mathcal{K}, D \rangle$	a matrix with designated set, D.
$\langle 0, 1 \rangle$	unit lattice structure with domain $[0, 1]$
$\langle 0, 1 \rangle_m$	finite lattice with domain $\{0, \frac{1}{1-m}, \cdots, \frac{m-2}{m-1}, 1\}$
v	a valuation function
$CT(\underline{\underline{\mathcal{M}}})$	content of $\underline{\underline{\mathcal{M}}}$
α^*	(pseudo) complement of α
$\overline{\alpha}\varphi$	signed formula
$S, \overline{\alpha}\varphi \mid \cdots \mid S, \overline{\beta}\psi$	at least one of the set of signed formulas is the case
$\mathcal{E} = \{U_1, \cdots, U_n\}$	a configuration
$T = \{\mathcal{E}_1, \cdots, \mathcal{E}_n\}$	a tableau
F_α	principal filter generated by α
I_β	principal ideal generated by β
$\wedge$	meet
$\vee$	join
$*$	complementation
$\neg$	strong negation
$\sim$	weak negation
$\underline{\underline{\mathcal{L}}}$	$G -$ logic
$\vdash_{\mathcal{L}}$	provability in $\underline{\underline{\mathcal{L}}}$

$F_\alpha I_\beta / f$	equivalence class in GK modulo f
GK/f	set of all equivalence classes GK modulo f
(X, τ)	hazy space
τ	hazy structure
ν	canonical hazy structure
ν_m	restriction of ν to $\{0, \cdots, m\}$
$\dot{\imath}$	indiscrete hazy structure
$\underline{m}$	the set $\{0, 1, \cdots, m-1\}$
$\nu(k)$	$\{k-1, k, k+1\}$, neighbourhood of k
$\tau(x)$	neighbourhood of x
$T = \{\tau(x) \mid x \in X\}$	neighbourhood system
$\sim_{(v,\tau)}$	indistinguishability relation w.r.t. v
$\sim_\tau$	indistinguishability relation
n	necessity function
p	possibility function
N_n	neighbouring necessity
N_p	neighbouring possibility
$\mathbf{n}$	necessity relation
$\mathbf{p}$	possibility relation
$\sim_I$	equivalence relation modulo I

$\lvert A \rvert$	cardinality of a (crisp or fuzzy) set
$\lvert A \rvert_F$	cardinal number of a fuzzy set as a fuzzy set
$\lVert A \rVert$	cardinal number of a fuzzy set as a natural number
$\mathcal{KS}(A, B)$	Kolmogorof-Smirnoff's degree of similarity
$\lVert v \rVert$	$\sum$-count validity coefficient
v_{ks}	Kolmogorof-Smirnoff's validity measure