

**In the Name of God**



Allameh Tabatabai University  
E.C.O. College of Insurance  
Actuarial Science

Master's Thesis

**The impact of determinants of mortality on life insurance and annuities**

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## *Abstract*

Recently extended risk classification has become an important issue in life insurance and annuity markets. Various risk factors have been explored and identified by past research. Using those risk factors, one can construct various risk classes. This enables insurers to provide more equitable life insurance and annuity benefits for individuals in different risk classes and to manage mortality/longevity risk more efficiently. The challenge of modeling mortality using various risk factors is to reflect complicated mortality dynamics in a model while maintaining statistical significance. This thesis discusses the development of a mortality model that reflects the impact of various risk factors on mortality. The model uses Markov process combined with generalized linear models, so that the model can reflect the nature of the determinants of mortality. Longitudinal survey data should be used for analysis and parameterization of the model and to determine the significant risk factors and risk classes and quantify their effect on mortality. The model is used to illustrate how the various risk factors influence actuarial present values of life insurance and annuity benefits.

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**Chapter 1:**  
**Introduction**

## **1.1. Motivation and Objectives**

For any type of insurance, premiums are charged to insureds based on the level of risk that is covered by a certain form of insurance product. Insurers normally use risk factors, which significantly affect specific risks that are related to the coverage they are offering. This allows them to classify a group of insureds according to the level of risk and to set different rates applicable to insureds in each risk class. As a result, policyholders are charged equitable premiums for insurance.

Specifically, mortality is a major risk inherent in life insurance, annuities, pension plans and other financial security programs. Age and sex have been used for more specific risk classification allowing actuaries to create mortality tables to set different rates for each age and sex group. For the last few decades smoking status have also been considered to be an important risk factor. Tables differentiating smokers from non-smokers are being used in pricing to reflect smokers' higher mortality risk (lower longevity risk).

In addition to the aforementioned factors, a great number of medical studies have explored and identified other significant mortality risk factors. This implies that mortality risk levels of the people in the same group based on the current risk classification can be extended if we take into account a larger number of risk factors. This displays two important motivations for actuarial work:

1. If more risk variables are included in an actuarial mortality model, more risk classes can be obtained. This means that an insurer can provide more equitable rates than the current system allows. As human life expectancy increases, largely due to the development of medical science, the economic security until death is becoming a very important issue. Specifically, the market for individual annuities providing retirement income is expected

to grow tremendously. In that case, more refined risk classification may be effective to reflect various levels of risk among annuitants to provide economic security with appropriate premium rates for insureds in each risk class.

2. The mortality assumption is one of the most important inputs for any actuarial valuation in life insurance, annuities and other financial security programs. The assumption affects the pricing and valuation process for risk management of insurance companies. An improved mortality model with additional risk factors will help to quantify the mortality rates for groups of insureds with increased accuracy. This allows the liabilities of insurers to be predicted more precisely. Failing to consider various risk levels among insureds may cause significant underestimation of mortality risk for certain groups of insureds and lead to insolvency of insurers.

So, construction of a mortality model with a variety of risk factors will provide benefits for both providers and participants in programs that assure mortality or longevity risks. This motivates the objectives of this thesis described as follows:

- 1. To explore risk factors affecting adult mortality**

There are various mortality risk factors identified by past studies. It is very important to know properties and behavior of risk factors before constructing a statistical model. Important risk factors will be explored by literature review.

- 2. To develop a mortality model with various risk factors and evaluate mortality differentials among risk classes**

A statistical mortality model will be developed with identified risk factors using Iran insurance company dataset. Mortality differentials among risk classes based on risk factors included in the developed model will be quantified.



### **3. To investigate the impact of mortality differentials on actuarial calculations**

The mortality differentials among the risk classes obtained by the model will affect actuarial calculations. The impact will be observed by numerous calculations of premiums of various life insurance and annuity products.

Even though this thesis focuses on mortality and its risk factors, the procedure of identifying risk factors, construction of a model with those risk factors and its application in this study provides a general approach to reflecting risk variables in other types of insurance.

## **1.2. Data Requirement**

Modeling mortality with larger number risk factors demands a proper data source that contains rich information on risk factors. The data must contain various risk factors and each risk factor should be classified by objective criteria for analysis. It's better to have a dataset which contain fairly frequent longitudinal observations for individuals to examine the behavior of risk factors over time.

Partly, the Iran insurance company dataset rather to other available dataset was found proper for development of a mortality model. Data are about individuals who participated in life insurance program. A subset of the data that includes only insureds over the age of 25 was extracted for analysis to focus on adult mortality in this study and they observed for 3 years (1384-1387). The data have some limitations. It doesn't include all of the risk factors that might be important in the case of mortality. Also there is no information about variability of risk factors over time.

So, we will construct the model by assuming that there is no change in the risk factors but the same approach can be applied in the case of variability of risk factors and inclusion of more risk factors.

Therefore, due to mentioned limitations, care should be exercised in using the results of our analysis for insurance applications.

### **1.3. Statistical Model**

As previously mentioned, the traditional actuarial approach for modeling mortality rates is to construct a mortality table according to age and sex. If additional factors such as smoking are considered, separate mortality tables are created. This approach is not feasible when a number of mortality risk factors are included in the model. Instead, we consider a statistical model that can accurately model mortality of various risk classes.

Mortality dynamics are very complex and difficult to explain exactly with a model. However, if we have more information on significant risk factors, the predictions will be significantly improved. There are important characteristics that the model should consider, the first being the interactions and correlations among various risk factors. Some of risk factors are highly correlated with each other. A generalized linear model is a useful tool both for explaining the impact of each predictor on a dependent variable as well as the effect of correlations among predictors. Because we need to quantify mortality rates, which are probabilities of death, as a function of risk factors and their interactions, logistic regression is considered to be a proper method. Unfortunately, if we classify risk states according to the identified risk factors, a risk state of an insured may change over time, moving the insured from one risk class to another. This transition can be modeled by logistic regression models as well.

Because there will be risk states that are time-dependent, we use a Markov chain as a framework for the model. Therefore, the mortality model will be a discrete time Markov chain with transition probabilities which are quantified by logistic regression models. The mathematical description of logistic regression and Markov chain will be provided in later chapters.

#### **1.4. Outline of Thesis**

As mentioned, the main goals of this thesis are to introduce a statistical mortality model, allowing for various risk factors of adult mortality, and show the application of such a model in actuarial science by investigating the impact of mortality differentials among risk classes on actuarial calculations for life insurance and annuities.

The mortality risk factors that have been considered to be important will be discussed in chapter 2, based on a literature review. As a starting point for the construction of a model, it is important to understand how each of the mortality risk factors affects mortality. Important results from previous studies dealing with mortality risk factors will be summarized. And the classification of risk factors according to the cause and effect relationships among them will be explained.

In chapter 3, we will introduce the generalized linear models and its special case logistic regression. In chapter 4, the theory of Markov chains and its application in actuarial science will be discussed and some of similar previous studies will be introduced.

The preliminary analysis of the Iran insurance company data will be presented in chapter 5. In this connection, the available risk factors in the data will be introduced and also the model for predicting mortality rates will be parameterized. In this chapter also, the application of the model

in actuarial science along with numerous calculations of premiums of various life insurance and annuity products will be presented. In chapter 6, the conclusion(s) and recommendation(s) for future research will be discussed.

**Chapter 2:**  
**Literature Review**

## **2.1. Mortality Risk Factors**

Human mortality is affected by countless factors with very complicated mechanisms. The main cause of death is deterioration of health which is largely due to the aging process and often accelerated by various kinds of diseases. Numerous risk factors that are relevant to mortality have been studied in previous medical epidemiological studies.

When dealing with the mortality for a specific age group, we should understand how each factor affects the mortality and also how those factors are related to each other. Brown and McDaid (2003) reviewed past studies regarding mortality risk factors at advanced ages. They identified 12 risk factors affecting mortality – age, alcohol, education, sex, health behaviors, income, marital status, obesity, occupation, race (ethnicity), religion and smoking. Those factors were found to affect mortality, causing the authors to suggest the use of extended risk classification using more risk factors in mortality assumptions for pricing annuities which are generally used as an income source after retirement. Modugno (2003) analyzed which of the factors that affect retirement mortality should be added to the existing mortality studies of the Society of Actuaries. Some factors were added or modified depending on availability from data sources. He suggested that mortality studies should include more comprehensive data including various risk factors for actuarial applications with extended risk classification. Methods of reflecting these factors in mortality tables were discussed and Recommendations were made for future mortality studies and to improve individual annuities for fairer pricing and to encourage insurance of longevity risk.

As mentioned there are several significant mortality risk factors. First of all, age and sex are the most fundamental determinants of mortality. Mortality risk increases with age due to the inevitable deterioration of body functions. Also there is a sex difference which is favorable for

females. These factors are fixed and interact with many other mortality risk factors. Other risk factors are mostly acquired and variable over time.

Socioeconomic status has been identified as one of the basic determinants of mortality. It is characterized income, education level or occupation of a person. Numerous recent studies have investigated the relationship between socioeconomic status and mortality. Sorlie et al. (1995) studied the effect of economic, demographic and social characteristics on mortality by observing a group of adults aged over 25. The study found higher mortality to be associated with lower income, less education and lower level of occupation groups. The effects of these socioeconomic factors for people at ages over 65 were weaker, but still significant. Furthermore, income was strongly associated with health behavioral risk factors. The authors argued that the reason for lower mortality of high socioeconomic group is their purchasing power for health services and preventive care, health conscious behavior, and useful health information that comes from education and their social relation. Bassuk et al. (2002) explored the effect of socioeconomic status on mortality in the community dwelling elderly (aged over 65) based on populations for four communities. The socioeconomic status was measured by income, education and occupational prestige. The authors found that socioeconomic status also affect mortality at advanced ages. Although the effect of socioeconomic status on mortality was attenuated after adjustment for health behavior, it was not eliminated.

Marital status is considered a significant predictor of mortality. Rogers (1995) investigated the effect of marital status on mortality considering sex and family income as covariates. The author focused on people aged from 25 to 64. As expected, lower mortality of married people was observed, and it was found that income mediates the relationship between marital status and mortality. That is married people are likely to have a high income, mostly due to dual income,

that leads to the economic advantages. There was also a sex difference inherent to the effect of marital status found by observing the interaction between sex and marital status. Men turned out to have a greater benefit from marriage than women.

Smoking is a well known risk factor that affects mortality through various adult diseases such as cancer and cardiovascular disease. This is widely used as a risk variable in mortality assumptions for many products provided by life insurers. The effect of smoking on mortality was explored and quantified by numerous past studies. Tessier et al. (2000) also studied the relation between smoking and mortality for the elderly population. In addition to confirming the significant effect of smoking on mortality, the authors found that intensity of smoking can differentiate mortality risk among smokers. The mortality risk factor of lifetime nonsmokers was similar to that of former smokers who had stopped more than 20 years ago. This indicates that the cessation of smoking produces long term effects on reduced mortality.

Drinking is another influential mortality risk factor. Klatsky et al. (1992) studied the relationship between alcohol intake and mortality with all age groups. They found that heavy drinking causes diseases such as cirrhosis and tobacco related cancers. It was also found that the effect of heavy drinking on mortality is higher for women and younger persons.

Regular exercise is one of the factors that improve health and mortality. It is widely known that physical fitness developed by regular activities reduces adult diseases by preserving health. Physical activity level may also indicate an individual's consciousness of health. Past studies explored how physical activity and fitness are related to mortality. Erikssen et al. (1998) investigated the effect of sequential changes in physical fitness on mortality with middle aged men. It was identified that even small improvements in physical fitness are associated with a significantly lowered risk of death. One interesting finding in the study was that physically fit



people are likely to be nonsmokers, which shows that physically active people are more health conscious than inactive people.

It is widely accepted that excessive body weight, largely due to body fat, causes various adult diseases. Body mass index, which is obtained by ratio of weight in kilograms to square of height in meters, is generally used to indicate the degree of body fat of an individual. Dorn et al. (1997) investigated the long term effects of body mass index on mortality from all causes and specific causes in the general population between 20 and 96 years of age. They observed a positive linear relationship between body mass index and mortality in men less than age 65 years, but not in women. The authors found that body mass index was strongly associated with cardiovascular disease and coronary heart disease mortality in women and younger men.

Blood pressure is one of the most important indicators of health outcome. Blood pressures that are too high or low are considered to be specific diseases themselves and to affect other critical adult diseases. Therefore, abnormal blood pressure levels significantly reduce the chance of survival. Lindholm et al. (1986) performed a study to investigate the relationship between mortality and blood pressure with a sample aged over 40. The blood pressure level was categorized by three groups: low, medium, and high. They found that the low and high groups of blood pressure showed higher mortality than the normal group in the ages 40 to 69. In those aged over 70, blood pressure appeared to be a less important mortality risk factor.

Cholesterol is another risk factor that should be considered. Health behaviors such as smoking, drinking, physical activity and dietary habit affect cholesterol levels. Anderson et al. (1987) explored the relationship between cholesterol and mortality with adults under the age of 65. They observed that cholesterol level is significantly associated with overall and cardiovascular mortality. That is, high cholesterol levels resulted in excess mortality. However they identified

that suddenly falling cholesterol levels increases mortality risk due to the critical diseases before death.

## **2.2 The Structure of Mortality Dynamics**

In addition to these factors that we mentioned, there may be other risk factors that affect mortality like family history of specific diseases, life satisfaction, stress level, depression, risk taking behavior, and.... These risk factors can be classified according to the cause–effect relationships among them. We consider the following three categories of risk factors:

- Socioeconomic/demographic risk factors: age, sex, education, income, occupation, marital status, etc;
- Behavioral risk factors: smoking, alcohol intake, physical activity, dietary habit, body mass index, etc;
- Health indicators: blood pressure, cholesterol level, blood sugar level, etc.

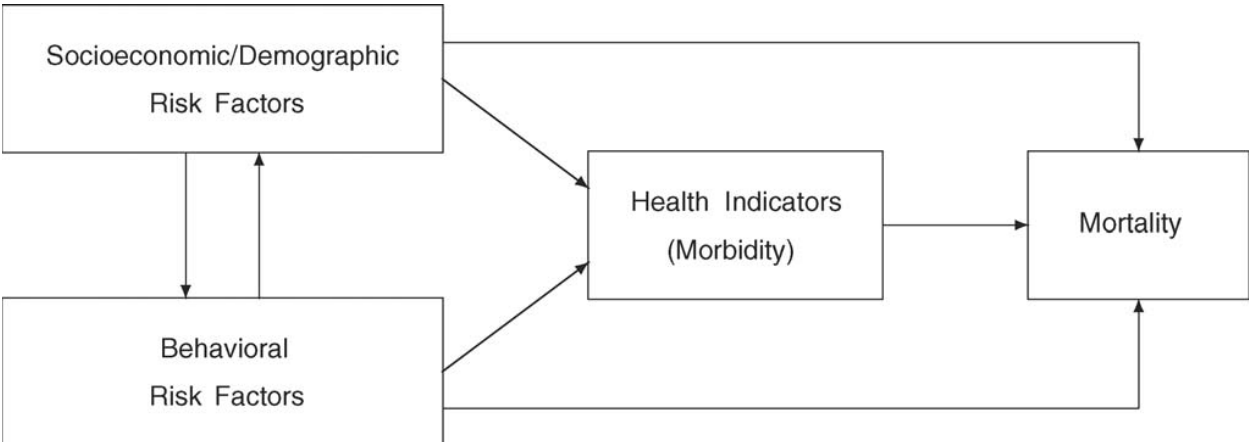
Socioeconomic/demographic risk factors are the fundamental determinants of mortality. These factors affect both behavioral risk factors and health indicators. Behavioral risk factors mediate the effects of socioeconomic/demographic factors on mortality. These two groups of factors determine health outcomes, which are normally measured and used to indicate specific diseases. Finally, mortality is directly influenced by health indicators. Also, mortality can be directly affected by socioeconomic/demographic variables (e.g. sex difference of mortality) and behavioral risk factors (e.g. accidental death caused by driving habits). The mortality dynamics are illustrated in Fig. 2.1.

Mortality is accelerated by incidence of any disease. Furthermore, major adult diseases are closely related to health indicators. Therefore, socioeconomic/demographic and behavioral risk

factors affect mortality through health indicators. This is supported by past research that found mortality risk factors affect major adult diseases. Health indicators can be used for short-term prediction of mortality, since people who have poor health have high mortality, which is obvious. However, if we focus on a long-term prediction of mortality using basic determinants of mortality, this will provide a more fundamental understanding of mortality.

In addition, policyholders at the issuance of the policy are normally healthy because of the underwriting criteria for insurability. Also, the durations of life insurance policies and annuities are fairly long. In this case, only long-term prediction of mortality using factors other than health outcomes remains. Therefore, investigation of the impact of socioeconomic/demographic and behavioral risk factors on mortality is very useful. This supports the use of a model with only basic determinants of mortality, especially for actuarial applications. Therefore, we try to explore an appropriate mortality model with available socioeconomic/demographic and behavioral risk factors.

Fig. 2.1. Mortality dynamics



## 2.3 Mathematical Models

In addition to the traditional survival analysis that has been popularly used in biostatistics, various mathematical models have been developed to explain mortality dynamics. Macdonald (1996) surveyed some statistical models of survival data. A basic model of a random life time was defined, and censoring was introduced. Markov and semi-Markov (multiple state) models were recommended as well-understood and flexible models well suited to actuarial data. He believed that the traditional Binomial-type models are more restricted than the modern multiple state models.

Macdonald et al. (2005a, b) introduced a model for the development of coronary heart disease (CHD) and they extended the model to include other critical illnesses, like cancers,... and described some applications of the model. They suggested a continuous time Markov chain with transition intensities presented by generalized linear models to explore the effect of risk factors on critical illness.

H.S. Kwon, and B.L. Jones (2006) described the development of a mortality model involving relevant risk factors and the use of this model to investigate the impact of these risk factors on actuarial calculations for life insurance and annuities. This paper introduced the Canadian National Population Health Survey (NPHS), the data set used in constructing the mortality model. A discrete time Markov chain is used to model transitions among the various risk factor states (and death), and transition probabilities are obtained using logistic regression models. The actuarial implications of the model are also presented in this paper.

As mentioned, we consider a mortality model to examine the effects and possible interactions of various risk factors. Such a model will allow us to better understand how the risk factors affect mortality and how they can be reflected in determining insurance and annuity premiums. The