CURRENT-VOLTAGE CHARACTERISTICS AND PARAMETRIC RESONANCE FEATURES OF COUPLED JOSEPHSON JUNCTIONS

by

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Dedicated to

My Family

Abstract

Charge formations on superconducting layers and creation of the longitudinal plasma wave in the stack of intrinsic Josephson junctions (IJJs) change crucially the superconducting current through the stack. The correlations of superconducting currents in neighboring Josephson junctions and the charge correlations in neighboring superconducting layers allows us to predict the additional features in the current–voltage characteristics. The charge autocorrelation functions clearly demonstrate the difference between harmonic and chaotic behavior in the so called breakpoint region. Use of the correlation functions gives us a powerful method for the analysis of the current–voltage characteristics of coupled Josephson junctions.

We study the phase dynamics in coupled Josephson junctions described by a system of nonlinear differential equations. Results of detailed numerical simulations of charge creation in the superconducting layers and the longitudinal plasma wave (LPW) nucleation are presented. We demonstrate the different time stages in the development of the LPW and present results of FFT analysis at different values of the bias current. The correspondence between the breakpoint position on the outermost branch of current voltage characteristics (CVC) and the growing region in time dependence of the electric charge in the superconducting layer is established. The effects of noise in the bias current and the external microwave radiation on the charge dynamics of the coupled Josephson junctions are found. These effects introduce a way to regulate the process of LPW nucleation in the stack of IJJ.

We calculate the CVC of IJJ and study the breakpoint region on the outermost branch of the CVC. A method for investigation of the fine structure in CVC of IJJ based on the recording the "phase-charge" diagrams is suggested. It is demonstrated that this method reflects the main features of the breakpoint region.

We demonstrate a manifestation of the charge traveling wave (TW) along the

c-axis in the CVC of the coupled Josephson junctions in high-T_c superconductors. The branches related to the TW with different wavelengths are found for the stacks with different number of Josephson junctions at different values of system parameters. Transitions between the TW branches and the outermost branch are observed. Time dependence of the electric charge in the superconducting layers and charge-charge correlation functions for TW and outermost branches show different behavior with bias current. We propose an experimental testing of the TW by microwave irradiation.

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Contents

Introduction

It is widely believed that the superconducting layers in high-T_c superconductors with large anisotropy are coupled by the Josephson effect. Several experimental results show the existence of the intrinsic Josephson effects in single crystals of high-T_c superconductors [1, 2, 3, 4, 5, 6, 7, 8]. For example it has been reported that the I-V characteristics along the c axis of Bi- 2212 in the superconducting state are those expected in a Josephson-junction array [2, 4]. The Fraunhofer pattern in the field dependence of the critical current has also been observed in those superconductors in a magnetic field parallel to the layers [1]. Recently, Matsuda *et al.* [7] observed a sharp resonance peak in microwave absorption experiments in single crystals of Bi-2212. This resonance peak has been identified with the Josephson plasma mode. These experimental results indicate that the electromagnetic properties of high-T_c superconductors are well understood on the basis of a model assuming a one-dimensional Josephson-junction array formed by superconducting layers of atomic-scale thickness. The mixed state of high-T_c superconductors has been intensively studied on the basis of such a Josephson coupled layered model [9].

Layered high-T_c superconductors can be considered as naturally stacked Josephson junctions (intrinsic Josephson junctions) [1, 2]. The dynamics of the superconducting phase differences in the intrinsic Josephson junctions have attracted a great deal of attention. It is well known that the phase differences in the intrinsic Josephson junctions are affected by the inductive coupling between junctions in the presence of a magnetic field and also by the charge-imbalance effect [10, 11] as well as the phonon effect [12, 13]. Furthermore, since the superconducting layers forming the junctions are extremely thin in the intrinsic Josephson junctions, a type of capacitive coupling appears between the junctions, which originates from the breakdown of the charge neutrality in thin superconducting layers as discussed in Refs. [14] and [15]. The capacitive coupling dominates the phase dynamics of the intrinsic Josephson junctions when no external magnetic field is applied and the charge imbalancing effect can be neglected. A new model which describes the phase dynamics of an array of capacitively coupled Josephson junctions (CCJJ) was proposed in Refs. [14] and [15]. The c-axis current – voltage characteristics of Bi-2212 under no external magnetic field have been successfully analyzed on the basis of this model [14, 15, 16].

About two decade has passed since the discovery of high-temperature superconductivity (HTS), but it still remains a challenge for physicists to derive a model that explains the main experimental features of the new superconductors. In the majority of theoretical models of HTS, it is assumed that the normal and superconducting state properties of the layered high-T_c cuprates derive mainly from the properties of the CuO_2 planes, while the other structural components in the unit cell act simply as charge reservoirs. The interlayer coupling has largely been considered as the mechanism of controlling the carrier concentration in the CuO_2 planes. The c-axis (or out-of-plane) coupling has become especially important since the interlayer tunnelling model of HTS has been developed [17]. In this model, the energy gain that drives the formation of Cooper pairs is associated with a decrease of the kinetic energy due to the easy motion of the pairs accompanied by the impeded single-particle tunnelling along the c-axis.

Another important question is whether the normal state out-of-plane transport is coherent or not, and what the origin of the 'semiconducting' c-axis resistivity in the cuprates is. Theories of out-of-plane transport differ in whether the zero-temperature state of cuprates is 'metallic' with a finite c-axis resistivity ρ_c, or insulating with an infinitely large $\rho_c(T \to 0)$. Major c-axis transport models have been examined in the review article by Cooper and Gray [18]. The physics of vortices has become one of the most quickly developing areas of modern physics. HTSs with their large anisotropy and thermal fluctuations represent a large research field where it has become possible to observe many new effects experimentally. A clear experimental evidence for melting of the classic Abrikosov vortex lattice in a wide temperature range below the superconducting critical temperature T_c [19] has verified a number of theories and stimulated further experimental and theoretical studies. The intrinsic Josephson effect (IJE) as a tunnelling of the Cooper pairs between adjacent CuO2 planes inside the highly anisotropic layered HTS is an integral part of many theories on this subject and is of primary importance for deriving properties of the vortex system [20]. It was experimentally confirmed by Kleiner and co-workers that the intrinsic tunnelling of the Cooper pairs indeed takes place in $Bi_2Sr_2CaCu_2O_{8+\delta}$ (Bi-2212) [1] and other anisotropic single crystals $Tl_2Sr_2Ca_2Cu_3O_{10+\delta}$ (Tl-2223) and $(Pb_yBi_{1-y})_2Sr_2CaCu_2O_{8+\delta}$ (Bi(Pb)-2212) [3].

In 1992 R. Kleiner and Müller demonstrated that high-T_c superconductors behave like a stacked array of Josephson junctions [1]. After that several models for stacked Josephson junctions were presented. Inductive coupling mechanism was introduced by Sakai, Bodin and Pedersen in 1993 [22]. Koyama and Tachiki suggested capacitive coupling in 1996 (Koyama-Tachiki model) [14]. Charge imbalance effect was described by Ryndyk [23] and phonon interaction effect by Helm *et al.* [12]. Shukrinov and Mahfouzi in 2006 phenomenologically derived the model which was called by them as the *capacitively coupled Josephson junctions model with diffusion current*(CCJJ+DC model). Their equations had a new term in comparison with CCJJ model, which was referred to as the *diffusion current.* Using this model they simulated the current-voltage characteristics of coupled system of JJs[25] and found good agreement with the experimental results. The same equations were derived by Machida *et al.* in 2000 by a microscopic theory (using Lagrangian of the system) [24]. But they considered the term corresponding to the diffusion current as being very small.

Some experiments were performed with single crystals of BSSCO by Irie *et al.* [26] and results of the experiments were compared with the results obtained by the CCJJ+DC model. A good agreement between experimental and theoretical results was demonstrated.

Detailed features of the breakpoint region were not investigated. Nucleation of longitudinal plasma wave also was not studied before. Chaotic features in the breakpoint region of the coupled system of intrinsic Josephson junctions were not investigated before. The charge traveling wave was also not investigated.

We established a correspondence between the features of current – voltage characteristics and the superconducting current in the breakpoint region. We investigated the superconducting current in the coupled system of Josephson junctions with longitudinal plasma wave (LPW) and clarified the role of the superconducting current correlations in different junctions, in the formation of the total current voltage characteristics. We demonstrated that the correlations of the superconducting currents in neighboring junctions and the correlations of the charge on superconducting layers manifest themselves as the features on the CVC, as a consequence of the phase dynamics in the breakpoint region. We showed that the correlation analysis is a powerful tool for the investigation of the CVC of the intrinsic Josephson junctions.

We found that the phase – charge diagram reflects the dynamical behavior of IJJ. It was shown that the results of the phase – charge diagram analysis are in agreement with the results of the autocorrelation function analysis. We demonstrated that using the phase – charge diagrams (phase portraits) give us a powerful method for investigation of the fine structure in the current – voltage characteristics.

We investigated the phase dynamics in the coupled system of Josephson junctions. We showed the different time stages of charge creation in S-layers and the development of the LPW. The data concerned the charge distribution along the stack and the results of the fast Fourier transformation analysis at different values of the bias current. It was found that the nucleation process of LPW can be affected by the noise in the bias current and the external microwave radiation. This makes it possible to regulate the process of LPW nucleation in HTSC.

We demonstrated the additional branches in CVC which corresponded to the charge traveling wave along c-axis (TW) when the junctions were in rotating states.

Such branches were not investigated before. Transitions in hysteresis region from the outermost branch (OB) to the traveling wave branch (TWB) and between the TWB with different wavelengths were found. We demonstrated the appearance of the TWB at I = Ic as well which originated from different branches of CVC including the zero-voltage branch. A different dynamics of correlation functions for the TWB and for the OB were shown. We found the effect of the microwave radiation on TWB, which allows the experimental testing of this branch.

We showed that CVC of coupled Josephson junctions has the branches related to the charge traveling waves along the c-axis. Transitions between such branches, demonstrating the changing of wavelength of the traveling wave, with a decrease of bias current were found. We found also the transitions from the outermost branch to the traveling wave branch. Detailed analysis of the time dependence of the charge in superconducting layer, its FFT analysis and the investigations of correlation functions showed different features of such transitions, particularly, different behavior for the outermost and traveling wave branches. These results shed light on the resonance features of the coupled Josephson junctions which are investigated intensively today. The detailed analysis of the branch structure in the experimental CVC of IJJ has not been performed yet. We proposed a method to distinguish the branch related to the charge traveling wave along c-axis from the other branches in CVC by microwave irradiation.

In this thesis we present the results of the investigation of the breakpoint region (BPR) structure in the framework of CCJJ+DC model. We use techniques such as : correlation function analysis, autocorrelation function analysis, analysis of Lissajous curves and analysis of charge-phase portraits. We investigate the effect of number of junctions on chaotic and regular parts of BPR. We investigate the chaotic behavior by analysis of Lyapunov exponent of the system. In the fifth chapter we show a novel type of branching in current – voltage characteristics of the system which corresponds to a traveling charge wave along the c-axis of stack.

The structure of this thesis is as follow:

In the first chapter I review some of the studies on research on coupled systems of Josephson junctions. I describe the basic properties of intrinsic Josephson junctions and their layered structure. I present a scanning tunneling microscope (STM) image of a typical intrinsic Josephson junction, the $Bi_2Sr_2CaCu_2O_8$. Then I describe different mechanisms of coupling and different models which are used for the investigation of the coupled systems of Josephson junctions: Inductive coupling, charge imbalance effect, phonon effect, capacitive coupling (Koyama – Tachiki model), and CCJJ+DC model. Then I describe the breakpoint region and its structure using the CCJJ+DC model. Next, I present some experimental results about the branch structure on the current – voltage characteristics and the breakpoint region. I also describe the resonance features in intrinsic Josephson junctions. The method of simulations concludes the chapter.

In the second chapter I describe the charge creation and nucleation of the longitudinal plasma wave in coupled Josephson junctions. First I show the current – voltage characteristics and time dependence of the charge in the superconducting layers. Then I distinguish some different time stages in the nucleation of longitudinal plasma waves in a stacked system of coupled Josephson junctions as : (I) Fluctuation region, (II) Island region, (III) Alternative amplitude region, (IV) Growing region. I present the effects of noise level and external radiation on the nucleation of longitudinal plasma waves.

I devote the third chapter to the investigation of the breakpoint region in current – voltage characteristic. The tools that I use for these investigations are: study of average of the supercurrent, study of current-current correlations, investigation of correlation of charge-charge, analysis of autocorrelation of charge. In the remaining part of this chapter I present the phase portraits of stacked Josephson array. Here I investigated the phase-charge diagrams.

In the fourth chapter I present an investigation of the chaotic features in the breakpoint region of current – voltage characteristics of coupled system. Here I show a good algorithm for calculating the Lyapunov exponent in coupled systems of Josephson junctions. I wrote a perfect program in *C language* and calculated the Lyapunov exponent by it. I presented the Lyapunov exponent of the linearized equation which were obtained by that program. Then I present the results with periodic boundary conditions and nonperiodic boundary conditions.

Then I wrote another program to calculate the Lyapunov exponent of the nonlinear equation. The results are shown in Chapter Four.

Fifth chapter is devoted to the demonstration of the traveling wave in the stacked Josephson junctions and new branches in current – voltage characteristics. I explain the origin of the traveling wave branch at near the critical current. Then I present the results related to the manifestation of traveling wave branch near breakpoint. Traveling wave branches with different wavelengths are presented and transition between traveling wave branches and the outermost branch are shown. In the next section I present the effect of the radiation. In the sixth section of this chapter I demonstrate the correlation function analysis of traveling wave branches.

Finally, in the last chapter I present the conclusions of my thesis.

Results of this thesis are published in the following papers and were presented in the following conferences.

Published papers

1) Yu. M. Shukrinov, M. Hamdipour, M. R. Kolahchi ,*"Effect of interjunction coupling on superconducting current and charge correlations in intrinsic Josephson junctions"* Physical Review B **80**, 014512, (2009).

This paper was also selected for publication in the Virtual Journal of Applications of Superconductivity , July 15, Volume 17, Issue 2, (2009).

2) Yu. M. Shukrinov and M. Hamdipour, *"Charge creation and nucleation of the longitudinal plasma wave in coupled Josephson junctions"*, Europhysics Letters (EPL), Volume 92 Number 3 , 92, 37010,(2010).

3) M. Hamdipour and Y.M. Shukrinov, *"Study of charge-phase diagrams for coupled system of Josephson junctions"*, Journal of physics: Conference series, **248**, 012042 (2010).

4) M. Hamdipour, Yu.M. Shukrinov, M. R. Kolahchi , *"Study of correlation and autocorrelation of supercurrent and charge in stacked Josephson junctions"*, Iranian Journal of Physics Research, **10** 2 155 , (2010).(Farsi).

5) Yu. M. Shukrinov and M. Hamdipour, *The c-axis charge traveling wave in coupled system of Josephson junctions*, arxiv:1103.5196v1.

6) M. Hamdipour, Yu. M. Shukrinov, M. Suzuki, M. R. Kolahchi, *"Transition from regular to chaotic dynamics in the intrinsic Josephson junctions"*, in preparation.

7) Yury Shukrinov, Ilhom Rahmonov, Mohammad Hamdipour, *"Simulation of Current Voltage Characteristics of Intrinsic Josephson Junctions in HTSC"* will be published in Springer as Lecture Notes in Computer Science.

Oral presentations and Posters in attended Conferences

1) Dubna International Advanced School of Theoretical Physics, IX Winter School on Theoretical Physics, NONLINEAR PHENOMENA IN CONDENSED MATTER, January 30-February 6, 2011, BLTP, JINR, Dubna, Russia.
2) XV Conferences of Young Scientists and Specialists 2011 AYSS'11, February 14-19, 2011, JINR, Dubna, Russia.
3) 2nd National Conference of Advances on Superconductivity, NCAS2, February 3-4, 2010 Ahvaz, Iran.
4) International Conference on Theoretical Physics, DUBNA-NANO2010, July 5-10, 2010, Dubna, Russia.
5) Mathematical Modeling and Computational Physics, July 4 - July 8, 2011, Star Lesn, High Tatra Mountains, Slovakia.

Chapter 1

Investigation of coupled Josephson junctions - Review

The basic equations governing the dynamics of the Josephson effect are

$V(t) = \frac{\hbar}{2e}\frac{\partial\varphi}{\partial t}$ (superconducting phase evolution equation)

$I(t) = I_c \sin(\varphi(t))$ (Josephson or weak-link current-phase relation)

where V(t) and I(t) are the voltage and current across the Josephson junction, $\varphi(t)$ is the "phase difference" across the junction (i.e., the difference in phase factor of the Ginzburg-Landau complex order parameter of the two superconductors composing the junction), and I_c is a constant, the critical current of the junction. The critical current is an important phenomenological parameter of the device that can be affected by temperature as well as by an applied magnetic field. The physical constant $h/2e$ is the magnetic flux quantum, the inverse of which is the Josephson constant.

The three main effects predicted by Josephson follow from these relations:

(I) The DC Josephson effect, this refers to the phenomenon of a direct current crossing from the insulator in the absence of any external electromagnetic field, owing to tunneling and with no voltage drop across the junction. This DC Josephson current is proportional to the sine of the phase difference across the insulator, and may take values between $-I_c$ and I_c.

(II) The AC Josephson effect, with a fixed voltage V_{DC} across the junctions, the phase will vary linearly with time and the current will be an AC current with amplitude I_c and frequency $\frac{2e}{h}V_{DC}$. The complete expression for the current drive I_{ext} becomes $I_{\text{ext}} = C_J\frac{dv}{dt} + I_J \sin\varphi + \frac{V}{R}$. This means a Josephson junction can act as a perfect voltage-to-frequency converter.

(III) The inverse AC Josephson effect, if the phase takes the form $\varphi(t) = \varphi_0 + n\omega t + a\sin(\omega t)$, the voltage and current will be

$V(t) = \frac{\hbar}{2e}\omega(n + a\cos(\omega t)), \quad I(t) = I_c \sum_{m=-\infty}^{\infty} J_n(a)\sin(\varphi_0 + (n+m)\omega t)$

The DC components will then be

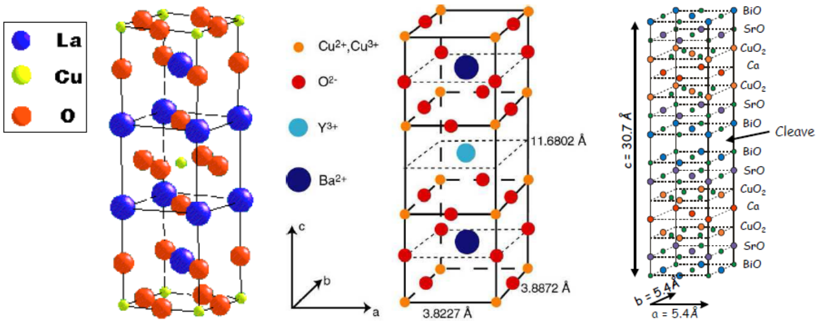
$V_{DC} = n\frac{\hbar}{2e}\omega, \quad I(t) = I_c J_{-n}(a)\sin\varphi_0$

Hence, for distinct DC voltages, the junction may carry a DC current and the junction acts like a perfect frequency-to-voltage converter.

Different models are used to investigate the IJJ. Here we first briefly explain the structure of some typical HTSCs. Then we turn to the different models which were developed recently and are used for modeling and simulation of IJJs. First model is the Sakai – Bodin – Pedersen model which includes the inductive mechanism of coupling [22]. Then the charge imbalance effect is described which was studied by J. Keller and D. A. Ryndyk [27]. In the third subsection the phonon effects are described [12, 11, 28, 29]. After that, we devote the fourth part to explanation of the Koyama – Tachiki model[14]. Finally, I describe the capacitively coupled Josephson junctions (CCJJ+DC) model[30]; this is the model which is mainly used in the thesis, and it will be explained in more detailed.

1.1 Basic properties of intrinsic Josephson junctions (IJJs)

As mentioned in the introduction, high temperature superconductors (HTSC) have a layered structure. Figure 1.1 shows the schematic structure of three high-T_c cuprate superconductor families: (a) $La_{2-x}Sr_xCuO_4$, (b) $YBa_2Cu_3O_{7-\delta}$, and (c) $Bi_2Sr_2CaCu_2O_{8+\delta}$. Also some experimental results show this layered structure more clearly. Figure 1.2 shows the scanning tunneling microscope (STM) image of BSCCO at 4K. In this picture we can see the CuO_2 double layers and space between them which has SrO and BiO layers.



(a) LSCO (b) YBCO (c) BSCCO

Figure 1.1: Structure of three high-T_c cuprate superconductor families: (a) $La_{2-x}Sr_xCuO_4$, (b) $YBa_2Cu_3O_{7-\delta}$, and (c) $Bi_2Sr_2CaCu_2O_{8+\delta}$.

The theory of layered oxide superconductors focuses on the possible type of coupling within the essential structure elements : the CuO_2 planes. However independent of the exact nature of pairing mechanism within the layers, the interlayer coupling determines most of the superconducting properties of a real crystal. In $Bi_2Sr_2CaCu_2O_8$ (BSCCO) the upper critical magnetic field is so large that the standard anisotropic Ginzburg-Landau theory yields c-axis coherence lengths ξ_c of the order of 1 $\mathring{A}$. This is much less than the distance of 12 $\mathring{A}$ between the CuO_2