Ferdowsi University of Mashhad

Faculty of Engineering

Department of Computer Engineering

Master's THESIS

By

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A New Type-II Fuzzy Logic Based Controller for Non-linear Dynamical Systems with Application to 3-PSP Parallel Robot

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Abstract

The concept of *uncertainty* is posed in almost any complex system including *parallel robots* as an outstanding instance of dynamical robotics systems. As suggested by the name, uncertainty, is some missing information that is beyond the knowledge of human thus we may tend to handle it properly to minimize the side-effects through the control process.

Type-II fuzzy logic has shown its superiority over traditional fuzzy logic when dealing with uncertainty. Type-II fuzzy logic controllers are however newer and more promising approaches that have been recently applied to various fields due to their significant contribution especially when noise (as an important instance of uncertainty) emerges. During the design of Type-I fuzzy logic systems, we presume that we are almost certain about the fuzzy membership functions which is not true in many cases. Thus T2FLS as a more realistic approach dealing with practical applications might have a lot to offer. Type-II fuzzy logic takes into account a higher level of uncertainty, in other words, the membership grade for a type-II fuzzy variable is no longer a crisp number but rather is itself a type-I linguistic term [29, 28, 17, 46, 31, 30].

Parallel robots on the other hand, are rather new sort of industrial and scientific tools that are being used in diverse research and industrial academia. The most problematic issues that engineers and designers face when using such robots are the high computational complexity needed for calculation of the inverse dynamics which should be recalculated at each iteration in the presence of structural uncertainty.

In this thesis the effects of uncertainty in dynamic control of a parallel robot is considered. More specifically, it is intended to incorporate the Type-II Fuzzy Logic paradigm into a model based controller, the so-called *computed torque control* method, and apply the result to a 3 degrees of freedom parallel manipulator.

One of the most well-known dynamic controllers that relies on the dynamic calculation of parameters of the underlying robot (in the feedback) is called the Computed Torque Control method. The CTC converts the non-linear dynamics of a robot into a linear one provided that the dynamics of the system at hand is completely identified. Having designed a system with a linear dynamic, it is easy for a control engineer to design a PID (or maybe PD) controller for it so that the final motion of the robot would be to follow a predetermined trajectory precisely.

The problem with the aforementioned method is that even if we manage to determine the foregoing parameters accurately we are yet to recalculate several matrices in each iteration. This imposes a high amount of computational burden. To overcome this demanding task, we should find a closed form formula for each of the dynamic terms so that not to perform intense computations that eventually leads to calculation of those parameters again and again.

Keywords: Robot Dynamic Control, Parallel Manipulator, 3PSP Robot, Type-II Fuzzy Logic, Computed Torque Control Method

Contents

List of Figures

List of Tables

CONVENTIONAL NOTATIONS

$\int$	Used to denote the union over elements of continuous fuzzy sets
$\sum$	Used to denote the union over elements of discrete fuzzy sets
$/$	Notation to separate a point in the domain with its membership grade
A, B, C	Widespread alphabets denoting T1 fuzzy sets
$\tilde{A}$, $\tilde{B}$	Widespread alphabets denoting T2 fuzzy sets
$\mu_{\tilde{A}}(.,.)$, $\mu_{\tilde{A}}(x,u)$	Another way to represent a T2 fuzzy set
$\mu_{\tilde{A}}(x',.)$, $\mu_{\tilde{A}}(x',u)$	Secondary membership function of a T2FS at point, x'
$\mu_{\tilde{A}}(x',u')$, $\mu_{\tilde{A}}(u = x', u = u')$	Secondary membership grade corresponding to domain point, x' with associated primary membership grade y' (note that sometimes the prime sign is omitted for convenience
$*$	T-norm
$\vee$	T-conorm
$\wedge$	Minimum t-norm
$\otimes$	A general binary operation
$\Im$	Used for denoting consecutive t-norms
$\prod$	Depending to the context, used for denoting both consecutive products and *meet* operation
$\coprod$	Used for denoting *join* operation

LIST OF ACRONYMS

FS	Fuzzy Set
FLS	Fuzzy Logic System
MF	Membership Function
T1	Type-I
T2	Type-II
T2FS	Type-II Fuzzy Set
T2FLS	Type-II Fuzzy Logic System
IT1FS	Interval Type-I Fuzzy Set
IT1FLS	Interval Type-I Fuzzy Logic Set
IT2FS	Interval Type-II Fuzzy Set
IT2FLS	Interval Type-II Fuzzy Logic Set
TR	Type Reduction
TG	Trajectory Generation
CTC	Computed Torque Control
SNR	Signal to noise ratio

Chapter 1

Literature Review, Type-II Fuzzy Logic

1.1 Preface

The knowledge to build a fuzzy logic system is itself uncertain, hence we expect our designed system to consider this uncertainty. This means that the output should not any longer be a crisp number. This is due to the fact that uncertainty has been propagated to the output as a result of uncertain information and inputs. Therefore the output should somehow represent this uncertainty. This is one of the most meaningful reasons why ordinary fuzzy logic systems (henceforth called T1FLSs) have given way to Type-II fuzzy logic systems (T2FLSs). In other words ([12]): increased fuzziness in a description requires increased ability to handle inexact information in a reasonably proper way. Uncertainty may come into existence from four main sources as follows [29]:

- The linguistic words being used in both antecedent and consequents of the fuzzy rules may mean different to different people.
- Some consequents derived from different line of thoughts of different experts, which may differ.
- These Noise interferences are in almost all of real-world applications.
- Also, the measuring devices which provide the inputs to fuzzy systems are themselves not exact and introduce noise to the inputs. [28], as follows:

Accordingly, in many situations we prefer to opt a Type-II fuzzy logic based approach the most appropriate instances of which are illustrated in [29] as follows:

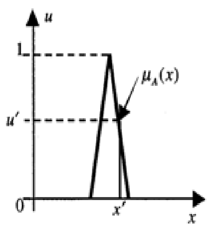
- ”Measurement noise is non-stationary, but the nature of the non-stationarity cannot be expressed mathematically ahead of time,” e.g., approximating a function under a variable SNR parameter.

- ”A data-generating mechanism is time-varying, but the nature of the time variations cannot be expressed mathematically ahead of time,” e.g., time variant communication channels.
- ”Features are described by statistical attributes that are non-stationary, but the nature of the non-stationarity cannot be expressed mathematically ahead of time,” e.g., rule-based classification of video traffic.
- ”Knowledge is mined from experts using IF-THEN questionnaires.”

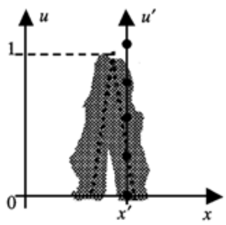
The question that strikes the mind of any newcomer to the realm of Type-II fuzzy logic is that why, despite the fact that the concept of Type-II fuzzy was introduced by Zadeh [46] in 1975, it was not welcomed and did not receive the attention of researchers, until the late nineties? In fact even now, the T2FL is considered as a state-of-the-art area of knowledge that has been emerging by introducing hundreds of valuable applications and supplementary theoretical techniques each year. One reason might be that science flourishes progressively. That is why old theories always give way to new ones and so forth. After introduction of T1FL by Zadeh in his seminal paper, no one understood or could foretell the future in progress; even in scientific academia some prejudicedly denied to accept it. It took some time until new applications of that theory became tangible to human and put it into practice. During the course of time scientists and more importantly engineers developed this logic until it became what it is today. The same history has been repeated for T2FL. That is to say, aside from few researches that considered the mathematical aspects of T2FL [7, 8, 9], it had not received much attention until the shortcomings of the predecessor became understood in a progressive way. Today we all know that the assertion ”Type-I fuzzy logic models uncertainty” that once was an undeniable fact, see for example paradox of Type-I fuzzy sets in [32, 21] , is not any longer believed to be true as they are inherently crisp and not fuzzy.

1.2 Type-II Fuzzy Sets

In this subsection we provide definitions for Type-II fuzzy sets (T2FSs) and the related but important concepts. This helps us to communicate effectively by laying a well-defined foundation for the language we use extensively through the rest of this thesis. Consider an ordinary fuzzy set depicted in Figure 1.1(a). What if we blur the portrayed membership function by shifting (not necessarily evenly) the points on the triangle up and down? Figure 1.1(b) shows the modified image. In this manner for a specific value in the domain, say , there will not be any unique membership grade attributed to it, rather the membership values now take on a continuous set of real values. This is illustrated in the latter figure. We can also ascribe a real value in [0,1] to each element of the aforementioned set to make up the third dimension. This second membership function is called *secondary membership function*, in literature.



(a) An ordinary fuzzy set



(b) having the membership function perturbed

Figure 1.1: Ordinary vs. perturbed fuzzy set. [34]

Doing so for all points in the domain, we come up with three-dimensional membership function which we call it a Type-II fuzzy set (T2FS). Among general Type-II fuzzy sets, two main groups are well-known and have been employed successfully, to date, namely the *Gaussian* Type-II fuzzy sets and *interval* Type-II fuzzy sets . The former includes Gaussian secondary membership functions while the latter includes interval valued Type-I secondary membership functions.

1.2.1 Gaussian Type-II Fuzzy Sets

Figure 1.2 depicts a 3-dimensional Gaussian Type-II fuzzy set in a 2D picture where the third dimension has been transferred to the image intensity. In the Figure, the darker points represent higher secondary membership grades and the solid line shows those points with unity secondary membership grade.

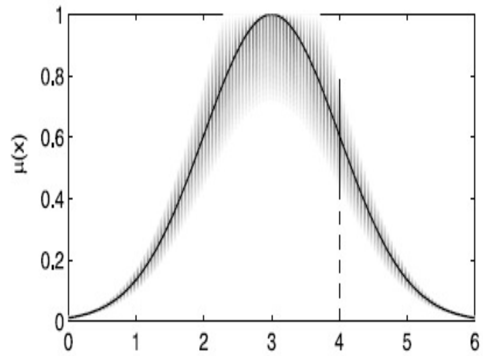


Figure 1.2: Type-II Gaussian fuzzy set portrayed from above view. [18]

1.2.2 Interval Type-II Fuzzy Sets

An important class of interval Type-II fuzzy sets (henceforth called IT2FSs) is Gaussian primary Type-II sets. Two different types of Gaussian primary T2FSs are: Gaussian IT2FSs with uncertain mean and Gaussian IT2FSs with uncertain standard deviation. Figures 1.3 illustrates these types of sets.

To communicate easier through the Type-II fuzzy literature we define the following important concepts, as well [34]:

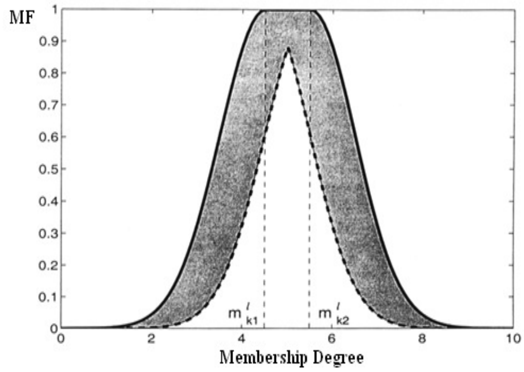
Definition 1.1 *A Type-II fuzzy set, denoted by $\tilde{A}$, is characterized by $\mu_{\tilde{A}}(x,u)$, where $x \in X$ and $u \in J_x \subseteq [0,1]$. That is to say,*

$$\tilde{A} = \{(x,u), \mu_{\tilde{A}}(x,u))|\forall x \in X, \forall u \in J_x \subseteq [0,1]\} \tag{1.1}$$

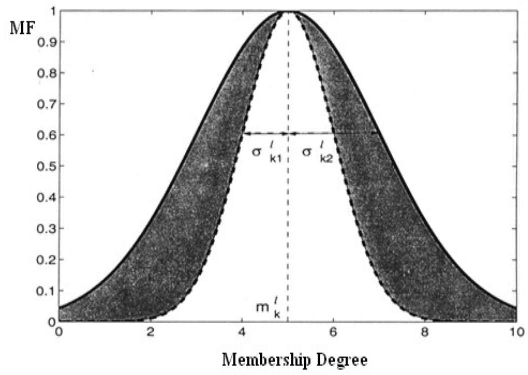
where $0 \leq \mu_{\tilde{A}}(x,u) \leq 1$. $\tilde{A}$ can also be re-expressed as

$$\tilde{A} = \int_{x\in X}\int_{u\in J_x} \mu_{\tilde{A}}(x,u)/(x,u) \quad J_x \subseteq [0,1] \tag{1.2}$$

where the integral sign represents the union over all admissible x, u. For discrete T2FSs $\int$ is replaced by the summation, Σ.



(a) Gaussian primary IT2FS with uncertain mean



(b) Gaussian primary IT2FS with uncertain sigma

Figure 1.3: Two types of Type-II Gaussian primary membership functions [22]

Definition 1.2 *The domain of the secondary membership is also called the* primary membership *and denoted by* J_x.

Definition 1.3 *At each value,* x'*, in the domain* X*, the 2D plane whose axes are* u *and* $\mu_{\tilde{A}}(x', u)$ *is called a* vertical slice *of the T2FS. Obviously this is a T1FS which can be stated as*

$$\mu_{\tilde{A}}(x = x', u) \equiv \mu_{\tilde{A}}(x = x') = \int_{u \in J_{x'}} f_{x'}(u)/(u) \quad J_{x'} \subseteq [0, 1] \tag{1.3}$$

Note that for convenience, it is common to use $\mu_{\tilde{A}}(x')$ *and* $\mu_{\tilde{A}}(x)$ *interchangeably. Note also that, as already stated* $\mu_{\tilde{A}}(x')$ *is the secondary membership function of a T2FS at* $x = x'$.

Definition 1.4 *The amplitude of a secondary membership is called a* secondary grade. *The secondary grade at* $x = x'$, $u = u'$ *is denoted by* $\mu_{\tilde{A}}(x = x', u = u')$.

Definition 1.5 *The union of all primary membership grades create a bounded region that we call the* footprint of uncertainty *(or FOU for short), i.e.,*

$$FOU\tilde{A} = \bigcup_{x \in X} J_x. \tag{1.4}$$

The shaded region in Fig. 1.3 is the FOU of the associated T2FS. The term FOU is very useful in T2 fuzzy literature in that it provides us with a convenient description of entire domain of the support *for all the secondary membership grades in a T2FS.*

Definition 1.6 *An interval Type-II fuzzy set (IT2FS) is a special kind of general T2FSs for which the secondary membership grades equal to 1. An IT2FS is completely defined by the footprint of uncertainty. These sets are the most widely used T2FSs due to several reasons (esp. the implementation issues, demonstrated in subsequent sections)*

Definition 1.7 *For a general T2FS,* $\tilde{A}$*, an* embedded Type-II set*,* $\tilde{A}_e$ *, is a T2FS such that for every admissible* $x \in X$ *in* $\tilde{A}_e$ *there is one and only one element, say* u_x*, in domain of* $\tilde{A}_e(x)$ *such that* $\tilde{A}_e(x, u_x) = \tilde{A}(x, u_x)$ *. For a discrete T2FS,*

$$\tilde{A} = \sum_{i=1}^{N} \sum_{j=1}^{M_i} \mu_{\tilde{A}}(x_i, u_{ij})/(x_i, u_{ij}) \quad J_{x_i} \subseteq [0, 1] \tag{1.5}$$

$\tilde{A}_e$ can be found as any of possible IT2FSs of the following form,

$$\tilde{A}_e = \sum_{i=1}^{N} \mu_{\tilde{A}}(x_i, u_i)/(x_i, u_i) \tag{1.6}$$

where $u_i \in M_i$. Accordingly for a discrete T2FS, there are a total of $\prod_{i=1}^{N} M_i$ different embedded T2FSs.

Definition 1.8 *For a T2FS, a T1 embedded FS is similar to T2 embedded one except that the secondary grade is now omitted so that the resulting set is in the form of a Type-I fuzzy set. For a general T2FS, $\tilde{A}$, this set is denoted as A_e and can be expressed (assuming the discrete case) as,*

$$A_e = \sum_{i=1}^{N} u_i/x_i \;\; u_i \in J_{x_i} \tag{1.7}$$

Definition 1.9 *When all the secondary membership functions corresponding to the domain of a Type-II fuzzy set are Type-I Gaussian membership functions, we call such a set a Gaussian Type-II set.*

Definition 1.10 *When all the secondary membership functions corresponding to the domain of a Type-II fuzzy set are interval Type-I membership functions, we call such a set an interval Type-II set (IT2FS).*

Definition 1.11 *A Type-II fuzzy set whose primary membership variable is discrete but its secondary membership functions are continuous is called a* partially discrete *Type-II fuzzy set. In the same manner, a Type-II fuzzy set whose both the primary variable and the associated secondary membership functions are crisp is called a* discrete membership *Type-II fuzzy set.*

Definition 1.12 *An* Upper *membership function (henceforth called* MF*) and a* Lower MF *are two Type-I MFs that are the upper and lower limits of FOU, respectively.*

1.3 Extension Principle

This principle extends the classical operations defined in crisp mathematics to the realm of fuzzy logic. In fact this is the basis of all T2FL analysis either explicitly or implicitly. The *principle states that if an operation $\otimes$, is defined on a set of numbers, then its counterpart in fuzzy domain is obtained by,*

$$F \otimes G = \int_u \int_w (f(u) * g(w))/(u \otimes w) \tag{1.8}$$

where the $\int$ sign and the $*$ denote the union and t-norm, respectively. In other words, if a relation $f(x)$, is defined on crisp variable $x \in X$ then we extend it to operate on fuzzy set, A, as follows:

$$\mu_B(y) = max_{x=f^{-1}(y)} \mu_A(x) \tag{1.9}$$

1.4 Operations in T2FLSs [18]

A T1FLS is a system that operates on T1FSs. Similarly a T2FLS is a system which manipulates T2FSs possibly along with T1FSs. These manipulations are mostly set-theoretic operations; however as is in this thesis, some systems perform algebraic operations too. The algebraic operations in T2FLSs are mainly performed on T1FSs which come out of the type reduction unit, as explained in upcoming sub-sections. Of course we can define algebraic operations on T2FSs, however normally they do not play a role in T2FLSs. For more details on such operations see [1]. Thus it is necessary to have a rather thorough discussion on both types of fuzzy operations.

1.4.1 Set-Theoretic Operations on Type-II Sets

Based on extension principle [46] the set-theoretic operations on T2FSs are declared as follows,

Union:

$$\tilde{A} \cup \tilde{B} \Leftrightarrow \mu_{\tilde{A}\cup\tilde{B}}(x) = \mu_{\tilde{A}}(x) \coprod \mu_{\tilde{B}}(x) = \int_u \int_w (f_x(u) * g_x(w))/(u \vee w) \tag{1.10}$$

Intersection:

$$\tilde{A} \cap \tilde{B} \Leftrightarrow \mu_{\tilde{A}\cap\tilde{B}}(x) = \mu_{\tilde{A}}(x) \prod \mu_{\tilde{B}}(x) = \int_u \int_w (f_x(u) * g_x(w))/(u * w) \tag{1.11}$$

Complement:

$$\overline{\tilde{A}} \Leftrightarrow \mu_{\overline{\tilde{A}}}(x) = \neg\mu_{\tilde{A}}(x) = \int_u f_x(u)/(1-u) \tag{1.12}$$