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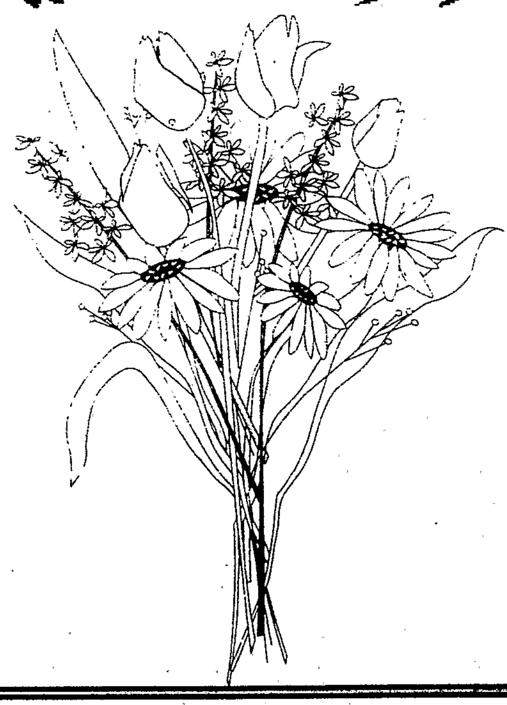

In the Name of Allah

The Optimal Control of an Inhomogeneous Wave and Heat Problem

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پیوست ....

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جلسه دفاع از پایان نامه آقای ابوالفضل علوی دانشجوی دورهٔ دکترای ریاضی درساعت ۹ صبح روز ۷۷/۱۱/۱۴ در اتاق شماره ۲۵ ساختمان خوارزمی دانشکده علوم ریاضی باحضور امضأ کنندگان ذیل تشکیل گردید. پس از بررسی واعلام نظر هیأت داوران، پایان نامهٔ نامبرده با درجه عالی مورد تأیید قرارگرفت.

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چکیده : (حداکثر ده سطر - لطفاً تایپ شود)

دراین رساله مسئله کنترل بهینه معادله موج غیر همگن باکنترل درونی بررسی شده است . ماباستفاده از تئوری نیم گروهها مسئله فوق را به یک مسئله گشتاور تبدیل کرده‌ایم وسپس آن را به مسئله کنترل بهینه درنظریه اندازه به یک مسئله برنامه ریزی خطی با بعد متناهی تقریب زده شده‌است وبه کمک جواب آن تابع کنترل بهینه تقریبی ومسیرهای مربوط به آن محاسبه شده‌اند.

مامسئله فوق را مشابهاً برای کنترل بهینه معادلات حرارت غیرهمگن باکنترل درونی نیز بررسی کرده‌ایم ما همچنین دراین رساله کاربرد تئوری اندازه را درحل معادلات دیفرانسیل معمولی وحل مسئله بولتزا دردو بعد استفاده شده است.

Abstract

In this thesis we consider an optimal control for an inhomogeneous wave problem with internal control and obtain its numerical solution. First we transfer the problem into an optimal moment problem. Then this problem is modified into the one consisting of the minimization of an infinite dimensional linear programming problem over a set of positive Radon measures. Finally we approximate the infinite dimensional linear programming problem to a classical finite dimensional one. The solution of this problem is used to construct an approximate piecewise-constant control. Also we study the controllability of the above problem in a special state and obtain exact optimal control for the problem.

Similarly, we consider the optimal control for an inhomogeneous heat problem with internal control and solve this problem by measure theory as well.

Also, we used measure theory to solve the optimal control for Bolza problem in two dimensions. We express that, under suitable assumptions optimal control is bang-bang and fined the approximate optimal control.

Finally, we introduce a new technique to find the approximate solution of a nonlinear ordinary differential equation by using measure theory, in this method we find the total error of the approximate solution.

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CONTENTS

Chapter 1

Preliminaries

In this chapter we study the basic concepts which are needed in the sequal

1.1 General background

Optimal control theory has been used in the solution of an enormous variety of problems in engineering, economic and biology. Many of the problem of design in airframe, shipbuilding, electronic, and other engineering fields are, in essence, problems of control ([5], [22] and [23]).

To define a classical control problem, we require to describe the components of the problem. To start the definitions we need, (i) a real closed time interval $I = [t_0, t_1]$, with $t_0 < t_1$, (ii) a bounded and closed subset $\mathcal{U}$ of R^m that in which the control functions take values, (iii) a differential equation describing the control system, satisfied by the trajectory function $t \in I \longrightarrow Y(t) \in R^n$ and control function $t \in I \longrightarrow u(t) \in \mathcal{U}$, where $u(t)$ is a measurable function, and (iv) an observation function $f_0(t, Y(t), u(t))$ which is assumed to be known. We can put further conditions on this function as necessary.

A classical optimal control problem is of finding an admissible control $u \in \mathcal{U}$ which satisfies the differential equation describing the controlled system and minimizes the functional $J : W \longrightarrow R$ defined by

$$J(p) = \int_I f_0(t, p)dt \ , \quad p \in W$$

where $p = (Y(\cdot), u(\cdot))$ and W is the set of admissible pairs of trajectories and controls.

Existence of optimal control and computation the approximation of an optimal control for the case where the system is controlled by ordinary differential equations considered by Lerner [34]-[37], Fel'dbaum [15]-[18] in their pioneering papers, Hestenes [26], Pontryagin et al [40], and Rubio [45]. Barbu [3] considered the problem for systems governed by variational inequalities.

The system whose state $Y(t) \in R^n$ is given by the solution of a partial differential equation combined with some appropriate boundary and initial conditions considered by many authors, we mention only Butkovskiy [5], Lions [38], Wang [55], Russel [48], [51]-[52], Egorov [9], Kamyad etal [27]-[28], Kamyad [29], Farahi [10]-[12]. In this thesis, we deal with the control theory of hyperbolic and parabolic partial differential equations, with internal control.

1.2 Semigroup Theory

The state-space theory of the control of lumped parameter systems is based on the solution of the following differential equation

$$x' = Ax \qquad x(0) = x_0 \in R^n$$

defined on n-dimensional Euclidean space R^n (where A is a matrix) and that of the perturbed form

$$x' = Ax + Bu.$$

These solutions are given, respectively, by

2.1 $$x(t) = e^{At}x_0$$

and

2.2 $$x(t) = e^{At}x_0 + \int_0^t e^{A(t-s)}Bu(s)ds.$$

In the case of distributed parameter systems, the mathematical description is usually given by a partial differemtial, we would like to be able to write the solution of a distributed system in the same form as (2.1) or (2.2).

1.2.1 Definition

A (strongly continuous) *semigroup* of operators is an operator-valued function $T : R^+ \longrightarrow \mathcal{B}(X)$ such that

a: $T(0) = I$,

b: $T(t_1 + t_2) = T(t_1)T(t_2)$,

c: $\lim_{t \to 0^+} T(t)x = x \qquad \forall x \in X$,

where $\mathcal{B}(X)$ is the space of all bounded operators defined on Banach space X (into X).

1.2.2 Definition

A closed operator A with dense domain in X is called the *generator* of the semigroup T if

$$\lim_{t \to 0^+} \frac{T(t)x - x}{t} = Ax \qquad x \in D \subset X.$$

where D is domain of A.

We now consider the inhomogeneous equation

$$2.3 \qquad x'(t) = Ax(t) + f(t) \qquad x(0) = x_0 \in X,$$

where A generates a semigroup $T(t)$ and f is continuously differentiable. If $x(t)$ is known to be a solution of (2.3), then

$$x(t) = T(t)x_0 + \int_0^t T(t-s)f(s)ds$$

This relation is called the *variation of constants formula* and shows that the solution of (2.1) is unique (see [4] Theorem 3.3.1).

1.2.3 Definition

Let $f : R \times X \longrightarrow Y$, for some Banach space X, Y. Then f is said to be *locally Holder continuous* in $t \in R$ and *locally Lipschitz* in $x \in X$, if for each $(t_0, x_0) \in R \times X$, there exists a neighbourhood U of (t_0, x_0) such that

$$\| f(t,x) - f(s,y) \|_Y \leq c_1 |t-s|^\alpha + c_2 \| x - y \|_Y$$

for all (t,x), $(s,y) \in U$, and some constants c_1, c_2, $\alpha > 0$.

We shall now consider the nonlinear equation

$$2.4 \qquad x'(t) = Ax(t) + f(t, x(t)) \qquad t > t_0, \quad x(t_0) = x_0,$$

where f is locally Holder continuous in t and locally Lipschitz in x and

$$\int_{t_0}^{t_0+t_1} \| f(s, x(s)) \| \, ds < \infty \qquad \text{for some } t_1 > 0,$$

then $x(t)$, the solution of (2.4), satisfies the integral equation

$$x(t) = T(t)x_0 + \int_{t_0}^{t} T(t-s) f(s, x(s)) ds \qquad t \in (t_0, t_0 + t1),$$

where $T(t)$ is a semigroup generated by operator A (see [4] Section 3.3).

1.3 Functionals and Measures

Let Ω be a closed subset of a finite-dimensional Euclidean space, in this section we shall consider spaces of functions and measures defined on a topological space, to be denoted also by Ω, which is locally compact and Hausdorff. (see [6] for a treatment of these concepts.)

We assume $C(\Omega)$ is the space of all continuous real-valued functions on Ω, with a topology defined by the norm

$$F \longrightarrow \|F\| = \sup_{z\in\Omega} |F(z)| \qquad F \in C(\Omega).$$

A *continuous linear functional* on $C(\Omega)$ is a linear function $\Lambda : C(\Omega) \longrightarrow R$ such that

3.1 $$|\Lambda(F)| \leq \lambda\|F\| \qquad F \in C(\Omega);$$

where λ is a constant independent of the particular function $F \in C(\Omega)$, but which will in general depend on the particular linear functional Λ. Let Γ be the set of all constants λ such that, for a fixed continuous linear functional Λ, (3.1) is true. Then

$$\|\Lambda\| = \inf \Gamma = \sup_{|F|\neq 0} |\Lambda(F)|/\|F\|$$

is a norm in $C(\Omega)^*$, where $C(\Omega)^*$ is the dual space of $C(\Omega)$.