In the Name of God

Razi University

Faculty of Engineering
Department of Civil Engineering

M.Sc. Thesis

Title of the Thesis

Development of Finite Element Method for 2D Numerical Simulation of Dam-Break Flow Using Saint-Venant Equations

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March 2012

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To my Parents

Abstract

The new computational methodologies are making numerical analysis more common in nearly all branches of engineering. In hydraulics and river engineering, an accurate numerical approximation of the governing equations in dam break type flows has recently become a major topic of interest. In this thesis, two-dimensional Saint-Venant calculations are presented by Finite Element Method (FEM). The employed procedure is formulated within the framework of Taylor-Galerkin scheme in conjunction with a Total Variation Diminishing (TVD) method, which is a Flux Limiter to reduce the spurious oscillations that emerge near sharp discontinuities. Despite the theme's significance in numerical studies, there has been very little investigation conducted on the scheme. Here, attempts were made to numerically investigate the capabilities of the proposed model, to simulate the wave propagation induced by dam break flows in a domain of two dimensions. Initially, proof is presented to authenticate the fact that the Saint-Venant Equations (SVEs) can be employed as the governing equations to numerically simulate the major aspects of a dam break flow. To do this, the derivation of SVEs is discussed, and the nature of equations is described in terms of eigenvectors and eigenvalues of the hyperbolic equations. A scheme of weak form is proposed to the SVEs using the Taylor-Galerkin method with a novel formulation while the topographic effects of the computational domain are ignored. Accordingly, the constructive method presented in the weak form will be used as a natural setting for a computational method to find the approximate solutions to the dam break flow. The numerical results of two different dam break type flows are presented; which are a 1D dam break and a 2D circular dam break problem. The proposed model is validated against the analytical and numerical solutions to water surface elevation and flow velocities, for the benchmark investigation cases. A decent harmony is observed, which indicates that the numerical scheme is capable of capturing the salient features of the respective flows in the numerical results. To thoroughly assess the capabilities of SVEs here, and the numerical scheme, the results acquired from the 2D circular dam break test case are schematically compared to the ones obtained from the simulation of the same problem using a well known CFD package that employs the Finite Volume Method (FVM) to solve the 3D Navier-Stokes and continuity equations.

Keywords:

Taylor-Galerkin
TVD
Flux Limiter
Saint-Venant Equations
Dam break flow

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List of Symbols

A,B	Constant coefficient matrices
A_i	Two dimensional integrations
F,G	Flux vectors
L	Characteristic mesh spacing
LA_i	One dimensional integrations
M	Mass matrix
N	Number all nodes
R	Known variables of the Dirichlet boundary conditions
S	Topographical and frictional source terms
U	Solution vector
a^i,b^i	Eigenvectors
b	Floor level
c	Celerity of gravity wave
f_c	Coriolis Parameter
g	Gravitational acceleration
h	Water depth
h_e	Element size
h_L	Water depth in the left hand side of the Dam
h_R	Water depth in the downstream channel
$\vec{k}$	The outward unit vector normal to Γ
n	Manning's roughness coefficient
p	Pressure
r^i,o^i	Eigenvalues
t	Non-negative variable representing time
u ,v, w	Velocity components
$\overline{u},\overline{v},\overline{w}$	Depth-averaged velocities
$\widehat{u},\widehat{v},\widehat{w}$	Velocities of the fluid particles in the free surface elevation
$\breve{v},\breve{u},\breve{w}$	Velocities of the fluid particles in the bed elevation
x	Horizontal coordinate
y	Transverse coordinate
z	Vertical coordinate
z_b	Bed elevation

Greek symbols

Ω	Computational domain
Γ	The boundary domain
Ψ	Shape function
α	Latitude
θ	Rotation rate of the Earth
ρ	Density
τ_{ij}	Viscous stress tensor

List of Acronyms

CBG	Characteristics Based Galerkin
CFL	Courant-Friedrichs-Lewy
CPU	Central Processing Unit
DWS	Diffusive Wave approximation of Shallow water equation
FDM	Finite Difference Method
FEM	Finite Element Method
FVM	Finite Volume Method
GAMBIT	Geometry And Mesh Building Intelligent
IHL	International Humanity Law
MATLAB	MATrix LABoratory
NPDP	National Performance of Dams Program
PDE	Partial Differential Equation
PISO	Pressure-Implicit with Splitting of Operators
RAM	Random Access Memory
RHS	Right Hand Side terms
RMSE	Root Mean Square Errors
SVE	Saint-Venant Equation
SWE	Shallow Water Equation
TVD	Total Variation Diminishing
TG- TVD	Taylor-Galerkin TVD
VOF	Volume Of Fluid

Chapter 1

Introduction

1.1 Motivation and Problem Statement

The new computational methodologies and developments in personal computers' CPU capacities are making numerical analysis more common in nearly all branches of engineering. In hydraulics and river engineering, two-dimensional (2D), depth-averaged models are starting to fasten together with one-dimensional (1D) models in general simulations. These models are useful in researches where local characteristics of velocity and depth distributions are significant. Instances include flood flow analysis, bridge design, and contaminant transportation.

The purpose of this thesis is the study of a shock-capturing technique applied to a finite element approximation of the Saint-Venant Equation (SVE) system, which is also referred to in the literature as the Shallow Water Equation (SWE) system. To conduct the computations, a new methodology is employed. The procedure is a combination of the classic framework of Galerkin method to discretize the equations in space, and the Taylor series expansion for the time discretization in conjunction with the Total Variation Diminishing (TVD) concept, to reduce the spurious oscillations that mainly emerge near abrupt changes of hydraulic flow properties (velocity, water depth, etc.), termed "discontinuities". The process is the scientific standard approach to Computational Fluid Dynamics (CFD) for compressible and shock-dominated flows. In this study, the system has been used to simulate the dam break flow comprising that the spillway is running throughout the failure.

We also carry out an investigation of the mathematical properties and derivation of SVEs, addressing the computational issues appropriate to conduct engineering problem solving procedures. To motivate more deeply this study, results obtained from the proposed Finite Element Method (FEM) are compared with the existing analytic and numerical solutions. This introduction is intended to present a condensed summary of the most important dam break phenomena and their calamitous affections, underlining the numerical modelling considerations and a brief review of previous studies.

1.2 Dam Break Phenomenon

Dam break flow is the instantaneous release of an initially stationary water body by removing a vertical obstacle; such as in case of a reservoir or a dam failure, the after effects transient flow over the bed is termed as dam break flow. When a dam is breached, calamitous flash flooding occurs as the impounded water flees through the opening into the downstream river. Generally, the response time available for warning is much shorter than that for acceleration of runoff floods. Consequently, the possibility of loss of lives is much deplorable. The emphasis on dam failure hazards have become well known enough that it is protected by the rules of International Humanitarian Law (IHL), and dams shall not be made the object of attack during armed quarrels if that may cause brutal losses amongst the civilian populations [1].

There have been around 200 important dam and reservoir failures in the world so far in the 20th century. Regardless of the affected dam, the resultant flow tend to cause damage to roads, railways, pipelines, power lines, telephone wires, houses and buildings, canals, drainage systems, bridges, ports, airports, forests and agricultural areas, which can become very serious. For instance, in August 1975 the world's most awful dam failure occurred in Henan[1] when the Banqiao Dam and the Shimantan Dam failed horrendously due to the overtopping caused by torrential rains. In this incident, around 85,000 people died because of the flooding, many more lost their lives during its consequent plague and starvation, and millions of occupants lost their houses. This disastrous is analogous to the events of Chernobyl and Bhopal for the nuclear and chemical industrial catastrophes.

The recent torrent in Pakistan in the summer of 2010, which was Pakistan's worst flooding in 80 years, has emphasized the saddening effect of flood flow on the lives of 20 millions of people along the floodplain of the River Indus. After all, the incidents underline the necessity to develop advanced flood-prevention measures to lessen possible flooding cause by dam breaks and other phenomena in the prospect.

Fig (1.1) shows the ten-year running average number of dam failures as available in the digital library of National Performance of Dams Program (NPDP) [2].

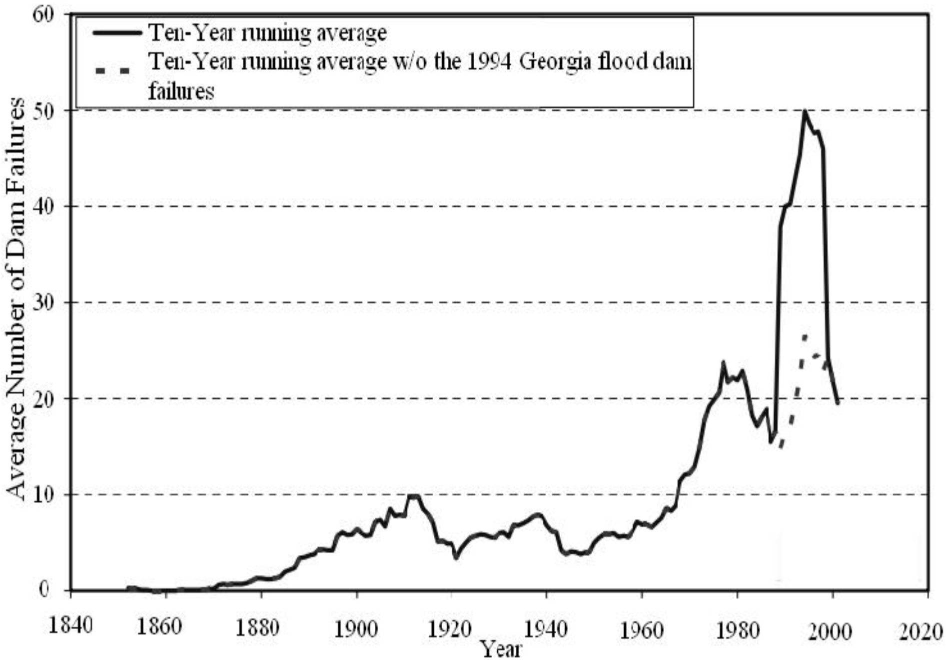

Figure 1.1: Ten-year running average number of dam failures (excluding the many small dams that failed during the 1994 Georgia floods) [2]

The running average is shown for the complete historic record (all failures) and for the record excluding the many small dams that failed during the 1994 Georgia floods.

Dams can fail for one or a combination of the following motives [3]:

- Overtopping due to floods that exceed the capacity of the dam
- Deliberate act of malicious damage or disruption
- Structural collapse of resources used in dam construction
- Movement and/or failure of the foundation supporting the dam
- Settlement and cracking of concrete or embankment dams
- Piping and internal erosion of soil in embankment dams

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[1] a province in China

- Unsatisfactory maintenance and upkeep

To reduce the effect of the calamity that these flows beget, certain considerations should be taken into account. Beside the attempts to eliminate the main causes and prevent the triggering, the first action to avoid catastrophe is to find solutions to mitigate the effects of the flood i.e. by moving the residents or infrastructures away from the areas of risk. Consequently, dam break study plays a prominent role when taking into consideration the reservoir safety, both for developing urgent situation tactics for existing structures and in considering the planning issues for new ones. The quick and ongoing improvement of computing power and procedures during the last 25 years has permitted significant promotes in the numerical modelling skills that could be functional in dam break analysis.

1.3 Numerical Analysis

In comparison to experimental investigations and because of the scale of the incident, numerical methods could be more attention getting to predict the flow behaviors, hydrographs, and routings. Numerical studies of currents in channels and rivers, mainly driven by floods, have an interesting range of applications in environmental hydraulics, navigation and provision of safe drinking water to the rural masses and industry use. It is evident that improved predictive means are required to appraise the likely impact of torrent and to contrive the emergency responses for particular areas. Consequently, the results obtained from the numerical simulations can play a major role in designs and further decisions. However, in mass dominated incidents; i.e. shock wave propagation in dam breaks, the problem cannot be solved using the classic approaches.

The dynamics of the shock wave propagation is rather complex and its behavior do not meet the terms of the regular assumptions of conventional steady and gradually varied open-channel flows. This fact is probably the major factor in the lag of investigations of dam break flows compared to coastal and estuarine problems. In order to comply with these constraints and address new computational challenges, the next generation of numerical models should be based on strong numerical techniques that accomplish the following set of standards [4].

- Inherent local and global conservation
- High-order accuracy
- Computational efficiency
- Geometric flexibility (any type of grid structure and suitable for adaptive mesh refinement)
- Non-oscillatory advection (Monotonicity-Preserving)
- High parallel efficiency

The numerical approaches developed for the related fluid dynamics problems, have been effectively adapted. These methods, referred to as “high resolution” or “shock capturing” schemes, allow generously combined different numerical methods to obtain precise results. For instance, the Taylor–Galerkin FEM presents excellent compromise among accuracy and CPU time of computations as it is based on Taylor series expansion up to the desired order [5]. To attain the previously mentioned requirements; in this thesis, a new scheme is shaped in terms of a Taylor series expansion in time. Using this approach, shock waves must be described for short time steps so the benefit of ease and speed of the explicit schemes gets more significant than the difficulty of conditional stability.

Although, there are several accessible commercial and free CFD packages to conduct the required hydraulic computations to simulate hydraulic phenomena; however, the packages are too expensive; moreover, each of the free ones is typically designed for a specific purpose. Being a closed-source program, it is unlikely to use a free software in a wide range of engineering problems; and as a result, learning the knowledge of numerical methods to code a specific solver for a particular phenomenon is inevitable. On the other hand, numerical simulations of phenomena like dam breaks or tsunamis, using physical illustrations such as Navier-Stokes equations, which are the default governing equations of the most CFD packages, can frequently be awkward due to the extent of the modelling geometries as well as through resolving free surfaces. However, SVEs of which there are a number of demonstrations, provide an easier picture of such phenomena.

In the current survey, all of these facets have been comprehensively evaluated. A survey of the literature exposes a motivating gap that there has been very little work on finite element based total variation diminishing method in unsteady currents, and especially in the dam break flows. This is rather strange, regarding the theme's significance in numerical simulations and accordingly as designing standards for floods, dam break waves etc., which are important topics in Civil Engineering.

1.4 Literature Review

In this subchapter, a short overview of the relevant literature is presented. The review is by no means comprehensive and is only relevant to the studies, carried out in this thesis. Accordingly, we will mainly consider those studies in which the usage of the SVEs in conjunction with numerical approximations and specially FEM was mainly effective in numerical investigations of meteorology, overland and dam break flow simulations.

Scott Russel's famous observation in 1834, of a Soliton [2] in the Union Canal in Scotland, and his experimental investigations of the phenomenon has significantly improved mathematical conception of the dynamics of nonlinear dispersive waves [6]. Restricting the attention to systems of equations; which admit travelling waves, for a smoother description of the dynamics of shock wave propagation in different domains and under certain boundary conditions, several suggested forms of SVEs can be found in literature. Shallow water systems can be deduced by an asymptotic expansion of the basic equations of fluid mechanics beginning from the Navier-Stokes equations and from appropriate boundary conditions [7]. This derivation founded on two assumptions, quite long waves or small height of the free surface comparing to the domain size in longitudinal direction, and the vertical velocity components are neglected so that the pressure distribution is hydrostatic. The numerical approximations of the SVEs are suitable for investigation of various physical problems and in many engineering applications, where reasonably lengthy waves occur. In fact, due to the special mathematical nature of the equations and the unique physical range, in which they can be employed, one cannot rely upon the mere transposition of schemes, which have been demonstrated to be successful for the SVEs [8]. Generally, dispersion is likely to necessitate that a great spatial precision and inertia term play a relevant role that asks for a proper description of the direction of propagation of the shock waves or other types of physical disturbances. Despite the common uses and thorough studies of the approximation of the SVEs by finite volumes and finite differences, the finite element discretization schemes are however for the most part not inspected.

[2] solitary wave

1.4.1 Numerical Approximation

Investigation teams have explored numerous models in order to capture the main features of flood currents in a computationally efficient manner. For instance, Xanthopoulos and Koutitas (1976) [9], as well as Hromadka et al. (1985) [10] validated a 2D dam break model for flood wave propagation and flood plain study, based on the diffusive wave approach. In their studies, they noticed that the inertial terms are insignificant in situations where the bed surface is flat, and derived the SVEs according to these conditions. They also concluded that the computational cost required solving the full SVEs increases by 50% the cost required to solve the Diffusive wave approximation of the SWEs (DSW). However, the frame is beyond the scope of this study.

Zhang and Cundy (1989), have developed an entirely dynamical model solving 2D SEVs, supposing specific types of the friction shear stresses by the use of a MacCormack Finite Difference Method (FDM) [11].Their model allows spatial variations of hill-slope attributes including surface roughness, microtopography, and infiltration. Their main conclusion was that microtopography is the principal factor causing variations in overland flow depth, velocity rate and path.

Frazao and Zech (2002) [12], conducted a series of numerical models and laboratory experiments of dam break flows in channels with 90^o bend with straight outlet reach. They solved the 1D and 2D SVEs in Finite Volume Method (FVM), and compared to the taken pictures of the water flow. They measured both the velocities in the bend and the water depth profiles along the channels and noticed that the 2D model was in a good agreement with the experimental data.

Ying et al. (2003) have also developed numerical models for flows generated by a dam failure or levee breaching process using the conservative form of SVEs, for more information refer to [13].

FEM and particularly in its most common technique, the standard Galerkin method, have been applied to the SVEs by the following investigators, to allude to but a few: Wang et al. (1972) [14] have employed the Galerkin method to solve the wave equations. They have concluded that the Galerkin scheme is more efficient than the ordinary FDMs. Baker (1973) [15, 16] has solved the problems involving inertia forces making use of Galerkin related methods. He has not solved problems with free surfaces and stress singularities. Cullen (1973) [17, 18] has applied a Galerkin method to simple two dimensional equations. The method is applied to passive advection problems and to a non-linear gravity wave problem useful in meteorological problems. Smith and Brebbia (1975) [19] have modeled a transient, incompressible viscous flow in two dimensions, where the dependent variables, stream function and vorticity, were approximated over each triangular element using linear interpolation functions. Brebbia and Partridge (1976) [20] have numerically simulated tidal effects, storm surges and currents in large bodies of water using six node triangular element and applied the model to the North Sea. Fang and Sheu (2001) [21] have merged Taylor-Galerkin and FCT methods, and applied it to several benchmarks e.g. partial dam break problem.

However, only very few finite element programs have been published that are designed to make it possible to sensibly employ the method to solve the SVEs. In meteorology, Wang et al. introduced use of the FEM, and the scheme has since then advanced significantly and that is now considered a tool preference by numerous researchers seeking to solve the 2D SVEs. Some common finite element approximations for solving hyperbolic equations are:

- Schemes for time integration with the development of higher order standard Galerkin approximation
- Method of least squares-Galerkin