IN THE NAME OF GOD

H_∞ OPTIMAL CONTROL AND SOME OF ITS APPLICATIONS

BY
MOHAMMAD GANJVAR

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EVALUATED AND APPROVED BY THE THESIS COMMITTEE AS:
EXCELLENT

M.MAHZOON, Ph.D., ASSISTANT PROF. OF MECHANICAL ENGINEERING (CHAIRMAN)

M.EGHTESAD, Ph.D., ASSISTANT PROF. OF MECHANICAL ENGINEERING

M. FARID, Ph.D., ASSISTANT PROF. OF MECHANICAL ENGINEERING

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Dedicated to
my

MOTHER & FATHER

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ABSTRACT

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H_∞ (pronounced as 'H-infinity') control theory has been an extensive research topic in the last ten years, and as a result, there are so many approaches to the state-space H_∞ control theory. An elegant proof was first given in Doyle, et al. 1989 [9].

The present work, which is basically an overview of the subject starts with a literature survey of H_∞ control theory that can be useful for researchers (Chapter 1). This follows by the preliminaries to understand the concepts (Chapter 2 and 3) and solving (Chapter 4) of H_∞ problem. Chapter 5 obtains the performance indices of optimal and robust control.

This relatively new approach to feedback design is explained through a solution for special case (state-space problem) given in Chapter 6. A better algorithm, however, and one which can be applied to general H_∞ problems, is given in Section 6.7 (without any derivation). Most of the contents are not new but we believe that the presentation given here may be of interest to those who are interested in learning H_∞ control with minimal mathematical background.

Finally, in Chapter 7 simple solved examples, illustrate this algorithm.

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ABBREVIATIONS

γ	H_∞ Bound or Lock Inertia Number
σ_i	i-th Singular Value
$\overline{\sigma}$	Maximum Singular Value
$\underline{\sigma}$	Minimum Singular Value
A, B…	Matrices (Bold Face Capital Words)
x, u…	Vectors (Bold Face)
e_{ss}	Steady-State Error
G	Plant
H	Hamiltonian Matrix
K	Generalized or Feedback System
L_2	Space of Signals with Finite 2-Norm
H_2	Subspace of L2 Composed on Analytic Functions in Re(s)>0
$H_2^\perp$	Subspace of L2 Composed on Analytic Functions in Re(s)<0
P	Generalized Plant
S	Closed-loop Sensitivity
t_f	Finite Time
T	Closed-loop Complementary Sensitivity
W_i	Weighting Matrices
Δ	Plant Uncertainty
$\|\cdot\|$	Vector Euclidian Norm
$\|\cdot\|_2$	Signal or System 2-norm
$\|\cdot\|_\infty$	System ∞-norm
$<\cdot,\cdot>$	Inner Product

CHAPTER 1

HISTORY AND LITERATURE SURVEY

Abstract

This part will very briefly review the history of the relationship between modern optimal control and robust control. The latter is commonly viewed as having arisen in reaction to certain perceived inadequacies of the former. More recently, the distinction has effectively disappeared. Once controversial notions of robust control have became thoroughly main streams, and optimal control methods permeate robust control theory. This has been especially true in H_∞ theory.

The primary focus of this chapter, which will serve as a short introduction to this thesis. Much of this is taken directly from [68], which also has most of the references.

1.1 The 70's, briefly

We could trace the origins of robust control almost arbitrarily far back in time, since robustness has always been the point of feedback, but we will start this narrative more recently by quickly recalling the controversy about LQG robustness in the mid to late 70's.One of the foundations of modern control theory was optimal control, which was tremendously successful in a variety of applications. The modern optimal control sample for feedback design, i.e. the LQG problem, however, had relatively little impact on practical control design. One of the critiques of LQG was that it did not directly address many issues that were already well understood in at least some limited way in classical control, and gain and phase margins were often pointed to as an example of this. The LQG proponents could, however, point to certain guaranteed properties of LQG regulators as indications of inherent robustness. As we now know, these guarantees were of no practical value, and indeed the whole notion of guaranteed margins is unacceptable.

At about the same time, singular values or the H_∞ norm was proposed for robustness analysis of multi-variable systems. This point of view added necessity to the small gain methods of the 1960s [65,63,52].

That is, whereas small gain gave sufficient conditions for stability for a set of uncertairty, the robust control interpretation was that the same condition was necessary and sufficient for a particular set. This emphasis on necessity motivated much study of the potential conservativeness of robustness measures and techniques for reducing it.

One of the motivations for the original introduction of H_∞ methods by Zames [66] was to emphasize plant uncertainty. The H_∞ norm was found to be appropriate for specifying both the level of plant uncertainty and the signal gain from disturbance inputs to error outputs in the controlled system. The H_∞ norm gives the maximum energy gain (the induced, L_2 system gain), or sinusoidal gain of the system. This is in contrast to the H_2 norm, which gives the variance of the output given white noise disturbances. The robust stability consequence was the main motivation for the development of H_∞

methods rather than the worst-case signal gain. We compromised on performance to get one norm that let us do everything. With this compromise, we could then talk about robust performance with structured uncertainty.

1.2 The 80's and the rise of H_∞

The synthesis of controllers that achieve an H_∞ norm specification gives a well- defined mathematical problem whose solution became a major research focus in the 1980s. Most of the original solution techniques were in an input-output setting and involved analytic functions (Nevanlinna-Pick interpolation) or operator-theoretic methods (Sarason [56], Ball- Helton[3]) and such derivations involved a fruitful collaboration between operator theorists and control engineers. Indeed, H_∞ theory seemed to many to signal the beginning of the end for the state-space methods, which had dominated control for the previous 20 years. Unfortunately, the standard frequency-domain approaches to H_∞ started running into significant obstacles in dealing with multi-input multi-output (MIMO) systems, both mathematically and computationally, much as the H_2 (or LQG) theory of the 1950's had.

Not surprisingly, the first solution to a general rational MIMO H_∞ optimal control problem [8], which we will refer to as the 1984 approach, relied heavily on state-space methods, although more as a computational tool than in any essential way. The procedure involved state-space inner/outer and coprime factorization of transfer function matrices, which reduced the problem to a Nehari/Hankel norm problem solvable by the state-space method in [17]. Both [14] and

[15] gave expositions of this approach, which in a mathematical sence "solved" the general rational problem. Much of the subsequent work in H_∞ control theory focused on the 2×2-block problem that was a central part of this solution, either in the model-matching or general distance forms. Unfortunately, the associated complexity of computation was substantial, involving several Riccati equations of increasing dimension, and formulae for the resulting controllers tended to be very complicated with high state dimension. Encouragement came from [34] and [35] who showed, for problems transformable to 2×1-block problems, that a subsequent minimal realization of the controller has state dimension no greater than that of the plant. This suggested the likely existence of similarly low dimension optimal controllers in the general 2×2 case. Additional progress came from [2, 27, 13, 24, 32, and 31].

Simple state-space H_∞ controller formulae were first announced in Glover and Doyle [18] (after some sustained manipulation). However the very simplicity of the new formulae and their similarity with the H_2 ones suggested a more direct approach. Independent enhancement for a simpler approach to the H_∞ problem came from papers by Khargonekar, Petersen, Rotea, and Zhou [28, 29]. They showed that for the state-feedback H_∞ problem one could choose a constant gain as a (sub)optimal controller. In addition, a formula for the state-feedback gain matrix was given in terms of an algebraic Riccati equation. Also, these papers established connections between H_∞ optimal control, quadratic stabilization, and linear-quadratic differential games. They showed that solving an algebraic Riccati