

IN THE NAME OF GOD

CALCULATION OF THE PROTON STRUCTURE FUNCTIONS IN  
NEUTRAL CURRENT DEEP INELASTIC SCATTERING  
ELECTRON + PROTON  $\rightarrow$  ELECTRON + X

BY

GHOLAM REZA BOROUN

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Evaluated and Approved by Thesis Committee As: Excellent

*Nader Ghahramany*

N.Ghahramany.Ph.D.,Associated Prof.of Physics

(Supervisor)

*A. Behkami*

A.Behkami.Ph.D.,Professor of Physics

*M.F. Rahimi*

M.F.Rahimi.Ph.D.,Associated Prof.of Physics

(Ferdowsi University)

*N. Riazi*

N.Riazi.Ph.D.,Professor of Physics

*M.H. Dehghani*

M.H.Dehghani.Ph.D.,Associated Prof.of Physics

*S.M. Zebarjad*

S.M.Zebarjad.Ph.D.,Assistant Prof.of Physics

July 2003

FAVEA

*Dedicated To My wife*

*For her patience, forbearance and encouragement*

ΕΛΥΣΑ

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## **Abstract**

### **Calculation of the structure functions in neutral current deep inelastic Scattering $e^- + p \rightarrow e^- + X$**

**By:** Gholam Reza Boroun / Elementary particle physics / July 2003

**Adviser:** Dr. Nader Ghahramany

High energy, i.e. deep-inelastic electron-proton collision, is the cleanest way to study the inner structure of the proton. The electron can probe the proton and its constituents via the electromagnetic interaction, much in the spirit of the famous Rutherford experiment which established the modern picture of the atom. The electrons thereby send virtual photons towards the proton scanning its internal structure like an electron microscope which renders the image of an object. The unknown proton structure is measured by two functions  $F_2$  and  $F_L$ , which correspond to different polarization states of the exchanged virtual photon. While the structure function  $F_2$  in a large part of the measured kinematic region is identical to the deep-inelastic scattering cross section itself the determination of  $F_L$  is experimentally much more challenging. The photons which mediate the electromagnetic interaction do not couple directly to electrically neutral gluons but to charged quarks. However the presence of gluons inside the proton can be felt by the struck quark. At large quark momenta  $x$ , quarks lose momentum by radiating gluons prior to the interaction with the virtual photon. Gluons can produce pairs of sea quarks which enhances the amount of quarks available for interaction with the proton at low momenta  $x$ . These processes can be resolved if the resolution of the photon probe, determined by its virtuality  $Q^2$  is sufficiently large. The amount of quark scattering partners is thus expected to increase with  $Q^2$ , i.e. at low  $x$  the structure function  $F_2(x, Q^2)$  should rise with  $Q^2$ , and this rise is determined by the gluon momentum distribution. These so-called scaling violations are indeed observed and they are well described by the theoretical calculation using QCD. In this thesis, a method to obtain an approximate relation between the reduced cross section derivative and the  $F_2(x, Q^2)$  scaling violations at low- $x$  is presented. The resulting formula can be used to determine the structure function  $F_2$  from the HERA data taken at low  $-x$ . This new approach can determine the structure function  $F_2$  with reasonable precision and an approximate analytical form of the gluon distribution function from the derivative of  $F_2$  proton structure function data at low  $-x$ . We use the Leading order A-P evolution equation in our analysis, and test its validity by comparing it with that of QCD analysis at low  $-x$  regime.

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# Chapter 1

## Structure Functions

### 1.1 Introduction

Scattering electrons off an atomic nucleus is a very effective way of investigating the electric charge and magnetization densities of the nucleus. The experiment is shown schematically in fig.1.1; an electron with four-momentum  $k = (E, \mathbf{k})$  scatters off the nucleus at rest and emerges at an angle  $\theta$  with momentum  $k' = (E', \mathbf{k}')$ .

The effective probe of the structure within the nucleus is the exchanged virtual photon with momentum  $q = k - k'$  with  $q^2 = -Q^2$  where  $Q^2 > 0$ . The resolving power of this probe is the *wavelength*  $\hbar/Q$  and so the degree of structure revealed increases with  $Q^2$  which depends on the energy  $E$  and the scattering angle  $\theta$ . In the lab frame, the energy of the virtual photon is  $\nu = E - E'$ .

For low values of  $Q^2$  ( $\sim 0.01 \text{ GeV}^2$ ) the nucleus tends to recoil as a