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BY

ALIREZA ASNAFI

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To:

My dear sons,

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ABSTRACT

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ALIREZA ASNAFI

Despite linear systems that admit general and unified approaches in their analysis and design, nonlinear systems do not in general lend themselves to general rules. During the past two decades, some researchers have tried to formulate general ideas for these systems using some tools from differential geometry. The invariant theme that exists in the language of differential geometry causes high level of generality and generality promotes the view points of system understanding, modeling and design.

In this thesis, using fiber bundle structure for the geometric approach, we try to find some unified rules in both forward and inverse dynamics problems for a variety of robotic locomotion systems. We show that once this structure is assigned to a locomotion system, we can talk about the gaits and behaviors that it may produce without numerically solving the dynamics of the overall system. Also we can design some unified open loop motion planner for both regulation and trajectory tracking problems.

Toward this goal we develop some unified formulas that relate the shape variables dynamics to the corresponding net displacement of a robotic locomotion system. Once these formulas are obtained we present a unified method to investigate the nonlinear behavior, find gait generation techniques and design motion planners. It is shown that in both symmetric and principally kinematic locomotion, these formulas are related to the components of the curvature of the connection while in mixed one, the components of locked inertia tensor and the scaled momenta must also be considered.

Using geometric approach, some merits of locomotion as compared to conventional mechanical systems are also studied.

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Abbreviations

∇	Affine connection
(ϕ_1, ϕ_2)	Fish-like swimmer and kinematic snake shape coordinates
(φ_1, φ_2)	Corresponding phases of (ϕ_1, ϕ_2)
$(\varphi_\psi, \varphi_\phi)$	Snakeboard's phases of the shape variables
Φ	Group action
ρ	The density of fluid and buoyant swimmer
ρ_a	Path radius of curvature
λ_i	Lagrange multipliers
ψ, ϕ	Snakeboard's shape coordinates
ω, ω_R	Constraint one-forms and reduced ω
$(\omega_\psi, \omega_\phi)$	Snakeboard's frequencies of the shape variables
(ω_1, ω_2)	Corresponding frequencies of (ϕ_1, ϕ_2)
ξ	The body representation of the Lie algebra element
ξ_Q	The infinitesimal generator
(a_0, b_0)	Location of the Center of the cycle produced by Shape variables in the base manifold
(a_1, a_2)	Corresponding amplitudes of (ϕ_1, ϕ_2)
(a_ψ, a_ϕ)	Snakeboard's amplitudes of the shape variables
A, A^{kin}, A^{nhc}	Mechanical, kinematic and nonholonomic connection

Ad	Adjoint map
D	Constraint distribution
F_i	Generalized forces
$DA, DA_{\Theta}, DA_U, DA_V$	Curvature of the connection and the rotational, longitudinal and lateral components of DA
G, g	Fiber manifold and the Lie group element
$\mathfrak{g}$	The Lie Algebra of g
h_i	Momentum like quantity
HQ	Horizontal space of Q
$\mathbf{I}$	Locked inertia tensor
I^{-1}	Local form of the locked inertia tensor
$I_S, (I_S)_U, (I_S)_V, (I_S)_{\Theta}$	Scaled inertia matrix and its components about (u,v, θ)
$\mathbf{J}$, $\mathbf{J}^{nhc}$	Momentum and nonholonomic momentum maps
J, J_r, J_w	Snakeboard's body, rotor and wheel inertias
$(I_{ij}^s, I_{ij}^f, I_{ij})$	Solid, added fluid and total inertial of the fish-like swimmer
I_{loc}	Locked inertia matrix of the fish-like swimmer
k_o, k_a	The ratio of frequencies and amplitudes of shape variables respectively
L, L_R	Lagrangian and reduced Lagrangian
M	Base manifold
M_{ij}	Metric tensor in base space
p, p^{nhc}	Generalized and generalized nonholonomic momenta
p_S	Scaled momentum