IN THE NAME OF GOD

STUDY OF NUCLEAR LEVEL DENSITY PARAMETER AND ITS TEMPERATURE DEPENDENCE IN FINITE NUCLEI

BY
MEHDI NASRI NASRABADI

THESIS
Submitted to the School of Graduate Studies is
Partial Fulfilment of the Requirement for the
Degree of Doctor of Philosophy(Ph.D)

IN
NUCLEAR PHYSICS
SHIRAZ UNIVERSITY
SHIRAZ, IRAN

Evaluated and Approved by the Thesis Committee As:
Excellent

A. Behkami, Prof. of Physics (Supervisor)
Shiraz University

S. Sobhanian, Prof. of Physics.
Tabriz University

M.R. Eskandari, Prof . of Physics
Shiraz University

A. Boshehri, Prof. of Chemsitry
Shiraz University

N. Ghahramany, Associate Prof .of Physics
Shiraz University

Feb. 2002

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Dedicated

To

my parents,

my wife who bore hard during the course of my research

and

my children Fatemeh and Mohammad

ACKNOWLEDGMENT

I would like to express my sincere gratitude and appreciation to my dear supervisor Professor Dr. A. N. Behkami for his help, guidance and encouragement during the course of my research. I would like to thank all my thesis committee members, Dr. Sobhanian, Dr. Eskandari, Dr. Boushehri, Dr. Ghahramany and Dr. Ghatee for their help and constructive suggestions in preparing this work. Finally I would like to thank all faculty members of college of science particularly Physics Department and Computer Center for their help.

ABSTRACT

Study of Nuclear Level Density Parameter and its Temperature Dependence in Finite Nuclei

By:

Mehdi Nasri Nasrabadi

The level densities of even - odd and even - even isotopes $^{161,162}Dy$, ^{166}Er and $^{171,172}Yb$ were calculated using microscopic theory of interacting fermions and is compared with experiment. It is found that, the data can well reproduced with level density formulizim for nuclei with static deformation. The nuclear temperature as well as the entropy of nuclear system as a function of excitation energy has been extracted from the BCS theory. It is shown that the entropy exhibits an S - formed shape as a function of excitation energy. This is interpreted as a phase transition. Procedure of treating the even - odd and even - even nuclear systems has been presented and discussed. Also using an exactly solvable pairing model Hamiltonian in BCS theory, we have analyzed the behaviour of *nuclear level density parameter* and its *temperature dependence* in *finite nuclei*. It is found that this important quantity varies linearly with A.

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Chapter 1

Introduction

The *nuclear level density* plays an important role in *nuclear reactions* such as the formation of the compound nucleus and the γ - decay rates of highly excited nuclei [1]. Thus the most relevant quantity describing the *statistical nuclear properties* is then the level density of the system.

Nuclear state densities have been the subject of investigation since the early days of nuclear physics and have led to an enormous number of experimental and theoretical papers. Reference to this work may be found in several reviews. see for instance. [2. 3. 4]. It would require a separate volume to summarize all the work have been done on this subject. which is outside of our scope. so here we will simply give a short account of the basic ingredients of the theory and will mainly discuss the expressions that are most frequently used in its calculation.

In what follows we make a clear distinction between *state* and *level*

densities, which will be indicated, respectively, by the functions ω and ρ of primary quantities such as the number of neutrons and protons (or more simply the mass number, A), the excitation energy, E, and the spin, J. Each level of spin J comprises $2J + 1$ degenerate states with different projection of J on the $z - axis$, so the relation between state and level density, considering that: $U = E - E_{ground}$, is

$$\omega(A, U, J) = (2J + 1)\rho(A, U, J) \tag{1.1}$$

The levels of a nucleus can be divided into *two energy regions*, namely the *low energy* and *high energy* excitations. This division arises naturally from the different approaches employed for their analysis: the *spectroscopical approach* for the low energy levels and the *statistical approach* for high energy levels. The low lying nuclear excited levels are small in number, well separated and rather simple in structure. For these levels the *spectroscopical approach* is the most suitable and leads to information concerning configurations, residual interaction and mixing.

The most outstanding feature of the total density of levels experimentally measured is its extremely rapid increase with excitation energy [5]. This is apparent in high-resolution experiments, where only at lowest excitation energies it is possible to resolve the peaks corresponding to the transitions to the discrete levels of the residual nucleus. At a few MeV of excitation energy these peaks partly overlap and then merge into con-

tinuum. This extremely rapid increase is characteristic of systems where the excitation energy is distributed among many degrees of freedom, as is to be expected in the nuclear case when several nucleons may be excited simultaneously.

The simplest expression for the nuclear level density has been obtained in the *fermi gas model* by Bethe [5, 6, 7] and later modified by Bloch [2, 8, 9]. As a zeroth order guess we assume that the distribution of states or level density is given by Bloch formula

$$\rho(E) = \frac{1}{\sqrt{48}E} \exp(2\sqrt{aE}) \tag{1.2}$$

where a is the *level density parameter*, often expressed in terms of the nuclear mass number A [10].

There are, however, a number of shortcoming in this approach. For example, the lack of coupling to the collective part of nuclear spectrum leads to an *energy independent level density parameter*. Recently, there has been considerable theoretical activity in the determination of nuclear many body density of states, taking into account *shell, pairing*, and *deformation effects* [11, 12], *finite size effects* [13], and *thermal* [14, 15] as well as improvements in the determination of the *spin cut off factors* [16].

Furthermore, the multiple inverse Laplace transform used to determine the nuclear density of states from the *Grand Canonical partition function* of Fermi gas appear to lead to certain inconsistencies in the fold-

ing of nuclear level densities [17]. In spite of these deficiencies Bloch's formula is widely used, particularly as a means of parameterizing the experimentally determined nuclear level densities [18]. In the present work we will demonstrate a simple means of extracting the nuclear level density from the experimental information, based on *BCS* (J. Bardeen, L. N. Cooper, J. R. Schrieffer) theory [19].

Anyway, since Bethe's first works on nuclear level density, many studies have been done for evaluation of this important quantity. But the familiar Bethe formula [5, 20] for the level density, because of its simplicity, has been widely used to perform the statistical analysis of nuclear reactions. This density formula takes a simple form due to (i): the connection of grandcanonical partition function with microcanonical partition by a saddle point approximation in the evaluation of the traces over the states [21] and (ii): the grandcanonical partition function itself is approximated using independent single particle spectrum which is further assumed to be equidistant [22].

In summary, the work on nuclear level density started by Bethe and expanded by widely using the partition function method. In its simplest form named Fermi gas model the nucleus was represented as a gas of noninteracting fermions confined to the nuclear volume [2, 5, 6, 7, 23, 24, 25, 26, 27, 28]. More specifically, the zeroth order expansion of this model was used, which corresponds to the equidistant model (equally

spaced energy levels). The equidistant model has been largely employed in data analysis and is very popular even at present although it contains little physical information.

A number of authors [29, 8] have presented a general procedure to include the shell model in level density calculations. However, much effort has been devoted to development of semiempirical approaches to the problem, either by modifying the parameters of the equidistant model formula [30, 31], or by introducing a shell correction in terms of an energy shift in ground state [32, 11]. A more fundamental attempt to understand the effect of shell model degeneracies has been made with the Rosenzweig degenerate model [33, 34]. In the same spirit, more sophisticated models based on schematic single particle level sequences have also been studied [35, 36, 37, 38, 39, 40, 41, 42]. However, such models lead to a relatively poor fit to experimental data because of their strong dependence on the choice of the single particle potential [43].

In an attempt to reproduce the experimental data, many modifications have been made to the original Bethe formula, considering the *shell, pairing* and *deformation effects*. This led first to the simple thermodynamic expressions which suggest an exponential dependence of the state density on the excitation energy U, namely

$$\omega(U) \propto \exp(U/T) \tag{1.3}$$

where T is the nuclear temperature [44], then to the shifted Fermi gas model and later to the popular back shifted Fermi gas model [11, 3]. The constant temperature formula has been widely used to analyse the spectra of particles emitted in statistical reactions and it reproduced the energy dependence of the emitted particle yield to an accuracy comparable with that obtained by use of more elaborate expressions of $\omega(U)$.

A number of authors showed that a constant temperature expression reproduces, at low energies, the experimental level densities better than the Fermi gas model [11], and recent analyses confirm its adequacy up to excitation energies around the neutron separation energy [45].

A more elaborate expression for the energy dependence of the state density is provided by the *equidistant spacing model ESM* which assumes that the one-particle states are equally spaced with spacing d. This model of the nucleus is clearly not realistic in many respects, since it neglects the residual interactions between the excited nucleons (so that the total energy of the nucleus is simply obtained by adding the energies of the constituent nucleons)and it gives highly degenerate excited states.

However, in a real nucleus the single particle states are split into a number of components by the residual interactions, and this is described by the imaginary part of the optical potential at negative energies. This fragmentation greatly increases the number of nuclear states. However, although this is indeed the case, it is not of practical importance if one is