In the name of Allah,

the beneficent,

the merciful.

دانشگاه شهید باهنر کرمان

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FUZZY Quality Control Charts

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A Dissertation Submitted as a Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in Mathematics (Ph. D.)

December 2008

دانشگاه شهید باهنر کرمان

دانشکده ریاضی و کامپیوتر

گروه ریاضی

رساله برای دریافت درجه دکتری ریاضی

نمودارهای کنترل کیفیت فازی

استاد راهنما:

دکتر ماشاالله ماشین چی

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آذر ماه ۸۷

Abstract

Control charts are the simplest type of on-line process control techniques. We consider the situation where a product unit may be classified into k ($k \geq 2$) different categories. The classical case has been studied by Duncan and Marcucci, among others, and Wang and Raz have suggested two kinds of control charts using probabilistic and fuzzy approaches. We propose yet another type of control chart that is more efficient in theory and practice.

In classical quality control, the binary classification of product units into "conforming" and "nonconforming" is used to construct the p-chart. When the quality characteristic is a variable, the p-chart, responds very weakly to small variations in the process mean and variance. In this thesis we consider quality as a fuzzy set and present a control chart in terms of the mean degree of nonconformity. We show that this chart has a better response to variations in the mean as well as the variance of process.

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Introduction

Control charts are widely used for monitoring and examining a production process. The power of control charts lies in their ability to detect process shifts and to identify abnormal conditions in the process. This makes possible the diagnosis of many production problems and often reduces losses and brings substantial improvements in product quality.

Linguistic scales are commonly used in industry to express properties or characteristic of products. Typically, the conformity to specifications on a quality standard is evaluated onto a two-state scales, e.g. acceptable or unacceptable, good or bad, and so on, which is not appropriate in many situations, where product quality can assume more intermediate states. Different procedures are proposed to monitor multinomial process when items are classified into k distinct categories. Duncan [6], recommended a chi-square contol chart for monitoring a multinomial process that is a generalization of the usual p-chart for which there are only two categories. This type of generalized p-chart is discussed further by Marcucci [18]. Marcucci proposed two procedures using Shewhart-type controls. The first type uses the Pearson χ^2 statistic and is designed to detect changes in any of the quality proportions. The second type uses the multinomial distribution, which can be approximated by a multivariate normal distribution. We shall discuss the generalized p-chart briefly in the next section.

Zadeh introduced the notion of fuzzy sets in 1965 [29] and has continued to discuss this concept, more recently in [30] and [31]. There have also been many efforts to apply the ideas of fuzzy sets to statistical problems [7, 14, 22, 24]. Raz and Wang [20] and Wang and Raz [25] proposed two approaches for the construction of control charts, namely a probabilistic approach and a membership approach. In the probabilistic approach, the representative values of the linguistic data were obtained by calculating the modes, medians, or fuzzy averages of their membership functions. These representative values were then utilized to construct control charts using traditional statistical methods. On the other hand, in the membership approach, the process level was estimated by the average of the total set of linguistic observations, and the representative value of the average was taken as the center line of the control chart. The control limits were then offset from the center line by a multiple of the fuzziness quantity of the estimated process level. Kanagawa *et al.* [15] introduced modifications to the construction of control charts given by Wang and Raz. Their study aimed at directly controlling the underlying probability distributions of the linguistic data, which were not considered by Wang and Raz. Kanagawa *et al.* [15] proposed control charts for linguistic data from a standpoint different from that of Wang and Raz in order not only to control the process average, but also to control the process variability. They presented new linguistic control charts for process average and process variability based on the estimation of the probability distribution existing behind the linguistic data. They defined the center line as the average mean of the sample cumulants and then calculated the control limits using Gram-Charlier series. The main difficulty of this approach is that the unknown probability distribution function cannot be determined easily. These procedures are reviewed by Woodal *et al.* [27]

and discussed by Laviolette *et al.* [17] and Asai [3]. Taleb and Limam [23] discussed different procedures of constructing control charts for linguistic data, based on the fuzzy sets and probability theories. A comparison between the fuzzy and probabilistic approaches, based on the average run length and the samples under control, is made using real data, and it is shown that contrary to the conclusions of Raz and Wang [20], the choice of degree of fuzziness affects the sensitivity of control charts. Gulbay and Kahraman [11, 12, 13] developed fuzzy approaches to control charts based on fuzzy transformation methods, which include fuzzy mode, fuzzy midrange, and fuzzy median. They used an α-cut approach to provide the ability of determining the tightness of the inspection (the higher the value of α the tighter inspection). They also presented a direct fuzzy approach to control charts. Chi-Bin Cheng [4] proposed the following approach to deal with the expert subjective judgments. Based on the rating scores assigned by individual inspectors to the inspected items, fuzzy numbers are constructed to represent the vague outcomes of the process. Then fuzzy control charts are constructed directly from these fuzzy numbers, thereby retaining the fuzziness of the original vague observations. The out of control conditions are formulated using possibility theory.

In this thesis, we generalize the p-chart to a fuzzy setting.

The organization of this dissertation is as follows:

In Chapter 1, we review the concept of the classical control charts and their OC curves. In Chapter 2, after recalling the fundamentals of fuzzy set theory we give a short survey of the generalization of these control charts to a fuzzy setting and in chapter 3, we propose yet another fuzzy generalizations. We use simulation to show that our chart is at least as effective as the previous charts, and in many cases, significantly more effective. Finally, we provide a conclusion.

Chapter 1

Quality control charts

1.1 Introduction

Statistical techniques in manufacturing and quality assurance have had a long history. In 1924 Walter A. Shewhart of the Bell Telephone Laboratories developed the statistical control chart concept. This is generally considered as the beginning of statistical quality control. Toward the end of the 1920s, Harold F. Dodge and Harold G. Romig, both of Bell Telephone Laboratories, developed statistically based acceptance sampling as an alternative to 100 percent inspection. By the middle of the 1930s, statistical quality control methods were in wide use at Western Electric, the manufacturing arm of the Bell System. However, the value of statistical quality control was not generally recognized by industry until World War II.

World War II saw the widespread use and acceptance of statistical quality control concepts in manufacturing industries. Wartime experience made it apparent that statistical techniques were necessary to control product quality. During World War II, Deming worked for the United States War Department and Census Bureau. Following the war, he become a consultant to Japanese industries and convinced their top management of the power of statistical quality control. This commitment to and use of these methods has been a key element in the expansion of Japan's industry and economy. Deming and others are creating in industry an awareness of statistics in general and statistical quality control in particular.

If a product is to meet the customer's fitness for use criteria, it should be produced by a process that is stable or repeatable. That is, it must be capable of operating with little variability around the target or nominal dimensions of the product's quality characteristics. Statistical process control (SPC) is a powerful collection of problem-solving tools use for achieving process stability and improving capability through the reduction of variability.

SPC can be applied to any production process. Its major tools are:

1. Histogram
2. Check sheet
3. Pareto chart
4. Cause and effect diagram
5. Defect concentration diagram
6. Scatter diagram
7. Control chart

Of these tools, the control chart is probably the most technically sophisticated. It was developed in the 1920s by Walter A. Shewhart of the Bell Telephone Laboratories. In order to understand the statistical concepts that form the basis of SPC, we must describe Shewhart's theory of variability [19].

1.2 Chance and assignable causes of quality variation

In any production process, regardless of how well designed or carefully maintained it is, a certain amount of inherent or natural variability will always exist. This natural variability or background noise is the cumulative effect of many small, essentially unavoidable causes. When the background noise in a process is relatively small, we usually consider it an acceptable level of process performance. In the framework of statistical quality control, this natural variability is often called a "stable system of chance causes". A process that is operating with only chance causes of variation present is said to be in statistical control. In other words, the chance causes are an inherent part of the process.

Other kinds of variability may occasionally be present in the output of a process. This variability in key quality characteristics usually arises from three sources: improperly adjusted machines, operator errors, or defective raw materials. Such variability is generally large when compared to the background noise, and it usually represents an unacceptable level of process performance. We refer to these sources of variability that are not part of the chance cause pattern as "assignable causes". A process that is operating in the presence of assignable causes is said to be out of control.

A major objective of statistical process control is to quickly detect the occurrence of assignable causes or process shifts so that investigation of the process and corrective action may be undertaken before many nonconforming units are manufactured. The control chart is an on-line process control technique widely used for this purpose. Control charts may also be used to estimate the parameters of a production process and, through this information, to determine process capability. The control chart may also provide information useful in improving the process. The eventual goal of statistical process control is the elimination of variability in the process. It may not be possible to completely eliminate variability, but the control chart is an effective tool in reducing variability as much as possible [19].

1.3 Statistical basis of the control chart

A typical control chart is shown in Figure1.1, which is a graphical display of a quality characteristic that has been measured or computed from a sample versus the sample number or time.

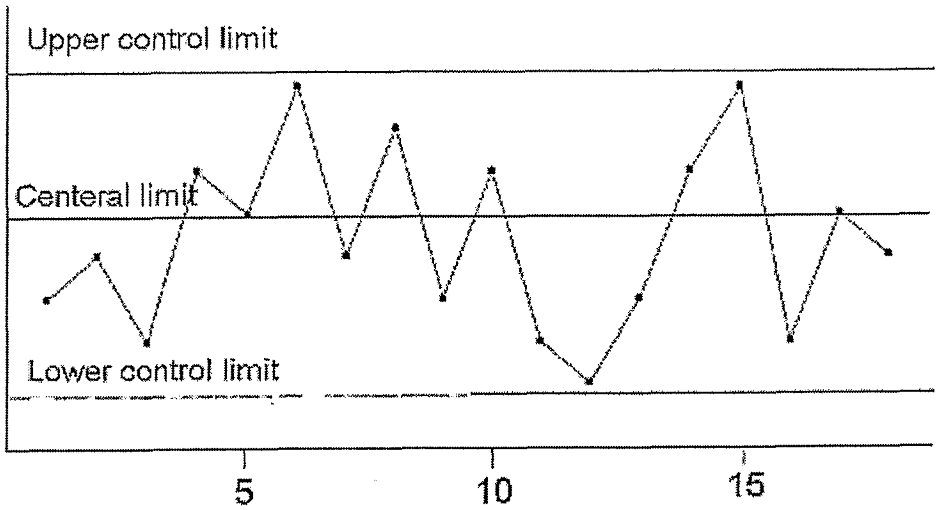

Figure 1.1: A typical control chart

The chart contains a center line that represents the average value of the quality characteristic corresponding to the in control state (i.e. when only chance causes are present). Two other horizontal lines, called the upper control limit (UCL) and the lower control limit (LCL), are also shown on the chart. These control limits are chosen so that if the process is in control, nearly all of the sample points will fall between them. As long as the points plot within the control limits, the process is assumed to be in control, and no action is necessary. However, a point that plots outside of the control limits is interpreted as evidence that the process is out of control, and investigation and corrective action is required to find and eliminate the assignable cause or causes responsible for this behavior. Even if all the points plot inside the control limits, if they behave in a systematic or nonrandom manner, then this is an indication that the process is out of control. If the process is in control, all the plotted points should have an essentially random pattern. Methods looking for sequences or nonrandom patterns can be applied to control charts as an aid in detecting out of control conditions. Usually, there is a reason why a particular nonrandom pattern appears on a control chart, and if it can be found and eliminated, process performance can be improved.

There is a close connection between control charts and hypothesis testing. Essentially, the control chart is a test of the hypothesis that the process is in a state of statistical control. A point plotting within the control limits is equivalent to failing to reject the hypothesis of statistical control, and a point plotting outside the control limits is equivalent to rejecting the hypothesis of statistical control. Just as in hypothesis testing, we may think of the probability of type I error of the control chart (concluding the process is out of control when it is really in control) and the probability of type II error of the control chart (concluding the process is in control when it is really out of control).

We may give a general model for a control chart. Let w be a sample statistic that measures some quality characteristic of interest, and suppose that the mean of w is μ_w and the standard deviation of w is σ_w. Then the center line (CL), the upper control limit (UCL) and the lower control limit (LCL) are defined as follows.

$$\begin{cases} UCL = \mu_w + k\sigma_w, \\ CL = \mu_w, \\ LCL = \mu_w - k\sigma_w, \end{cases} \tag{1.3.1}$$

where k is the "distance" of the control limits from the center line, expressed in standard deviation units. This general theory of control charts was first proposed by Walter A. Shewhart, and control charts developed according to these principles are often called Shewhart control charts.

The most important use of a control chart is to improve the process. We have found that, generally:

1. Most processes do not operate in a state of statistical control.

2. Consequently, the routine and attentive use of control charts will identify assignable causes. If these causes can be eliminated from the process, variability will be reduced and the process will be improved.

3. The control chart will only detect assignable causes. Management, operator, and engineering action will usually be necessary to eliminate the assignable cause.

In identifying and eliminating assignable causes, it is important to find the underlying root cause of the problem and to attack it. A cosmetic solution will not result in any real, long-term process improvement. Developing an effective system for corrective action is an essential component of an effective SPC implementation. We may also use the control chart as an estimating device. That is, from a control chart that exhibits statistical control, we may estimate certain process parameters, such as the mean, standard deviation, fraction nonconforming or fallout, and so forth.

Control charts may be classified into two general types. If the quality characteristic can be measured and expressed as a number on some continuous scale of measurement, it is usually called a variable. In such cases, it is convenient to describe the quality characteristic with a measure of central tendency and a measure of variability. The $\bar{x}$-chart is the most widely used chart for controlling central tendency, while charts based on either the sample range or sample standard deviation are used to control process variability. Many quality characteristics are not measured on a continuous scale or even a quantitative scale. In this case, we may judge each unit of product as either conforming or nonconforming on the basis of whether or not it possesses certain attributes, or we may count the number of nonconformities appearing on a unit of product. The p-chart is used to monitor the fraction of nonconforming units, and the c-chart is used for the number of nonconfomities per product unit [19].

1.4 Choice of control limits

Specifying the control limits is one of the critical decisions that must be made in designing a control chart. By moving the control limits further from the center line, we decrease the risk of a type I error, that is, the risk of a point falling beyond the control limits, indicating an out of control condition when no assignable cause is present. However, widening the control limits will also increase the risk of a type II error, that is, the risk of a point falling between the control limits when the process is really out of control. If we move the control limits closer to the center line, the opposite effect is obtained: the risk of type I error is increased, while the risk of type II error is decreased.

Regardless of the distribution of the quality characteristic, it is standard practice to determine the control limits as a multiple of the standard deviation of the statistic plotted on the chart. The multiple usually chosen is k=3 (α=0.0027). Hence, 3-sigma limits are customariliy employed on control charts, regardless of the type of chart employed. In the United Kingdom and parts of Western Europe, k is 3.09 (α=0.002).

We typically justify the use of 3-sigma control limits on the basis that they give good result in practice. Moreover, in many cases, the true distribution of the quality characteristic is not known well enough to compute exact probability limits. If the distribution of the quality characteristic is reasonably approximated by the normal distribution, then there will be little difference between 3-sigma and 3.09-sigma limits [19].

1.5 Analysis of patterns on control charts

A control chart may indicate an out of control condition either when one or more points fall beyond the control limits, or when the plotted points exhibit some non-random pattern of behavior. The Western Electric Handbook [26] suggests a set of decision rules for detecting nonrandom patterns on control charts. Specifically, it suggests concluding that the process is out of control if either:

1. One point plots outside the 3-sigma control limits.

2. Two out of three consecutive points plot beyond the 2-sigma warning limits.

3. Four out of five consecutive points plot at a distance of 1-sigma or beyond from the center line.

4. Eight consecutive points plot on one side of the center line.

1.6 The operating-characteristic function and average run length

The ability of charts to detect shifts in process quality is described by their operating characteristic (OC) curves. The OC function is a graphical display of the probability of incorrectly accepting the hypothesis of statistical control (the probability of type II error) against shifts in process quality. The average run length (ARL) is the average number of points that must be plotted before a point indicates an out of control condition. For any Shewhart chart, the ARL can be calculated as follows:

$$ARL = \frac{1}{\text{probability that one point plots out of control}}\,. \tag{1.6.1}$$

Thus, if the process is in control, the ARL is

$ARL = \frac{1}{\alpha}$

and if it is out of control, then

$ARL = \frac{1}{1-\beta}$

For any Shewhart chart with the usual 3-sigma limits, assuming normality p=0.0027 is the probability that a single point falls outside the control limits when the process is in control, so $ARL = \frac{1}{0.0027} = 370$, when the process is in control. That is, even if the process remains in control, an out-of-control signal will be generated every 370 samples, on the average [19].

1.7 The control chart for fraction nonconforming

Many quality characteristics cannot be conveniently represented numerically. In such cases, we usually classify each item inspected as either conforming or nonconforming to the specifications on that quality characteristic. The terminology "defective" or "nondefective" is often used to identify these two classifications of product. More recently, the terminology "conforming" and "nonconforming" has become popular. Quality characteristics of this type are called attributes. The fraction nonconfoming is defined as the ratio of the number of nonconforming items in a population to the total number of items in that population. The items may have several quality characteristics that are examined simultaneously by the inspector. The underlying statistical principles for a control chart for the proportion of nonconforming units are based on the binomial distribution. Suppose that the production process operates in a stable manner, such that the probability that a given unit will not conform to specifications is p, and that successive units produced are independent. Then each unit produced is a realization of a Bernoulli random variable with parameter p. If a random sample of n units of product is selected and if D is the number of units that are nonconforming, then D has a binomial distribution with parameters n and p, that is,

$$\begin{aligned} D &\sim Bin(n,p), \\ E(D) &= np, \\ Var(D) &= npq. \end{aligned} \tag{1.7.1}$$

The sample fraction nonconforming is defined as the ratio of the number of nonconforming units in the sample D to the sample size n, that is, $\hat{p} = \frac{D}{n}$. $\hat{p}$ is an estimator of p and the mean and variance of $\hat{p}$ are $\mu_{\hat{p}} = p$ and $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$.

Suppose that the true fraction nonconforming p in the production process is known or is a standard value specified by management. Then from (1.3.1), the center line and control limits of the fraction nonconforming control chart would be